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Please fill in your manuscript title.	A brief summary of seventy years of well test analysis

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Given Name	Surname	Company
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Abstract

Objectives/Scope: Transient well test analysis has been used to assess well condition and obtain reservoir parameters for over seventy years. At a time when Machine Learning (ML) and Artificial Intelligence (AI) are coming-of-age and may eventually replace well test interpreters, it is worth taking stock of the progress made in well test analysis since the major initial publications of the early 1950's.

Methods, Procedures, Process: Major improvements in well test analysis since the early 1950's have occurred approximately 13 to 19 years apart, driven by the availability of both new types of data and new mathematical tools. Using a computer-generated data set of pressure and rate, we illustrate the evolution of well test analysis over the years: straight line methods of the early 1950's; log-log pressure plots of the late 1960's and early 1970's; formulation of an integrated methodology in the early 1980s; introduction of pressure-derivative analysis in 1983; derivation of a stable deconvolution algorithm in the early 2000's; and its current, successful extension to multiple interfering wells.

Results, Observations, Conclusions: Over the last seventy years, well test analysis has moved from just estimating well performance to becoming a very powerful tool for reservoir characterization.

Novel/Additive Information: Although the development of commercial software has been a factor in making practicing engineers aware of these improvements, acceptance has been slow: deconvolution has been developed twenty years ago and has been included in commercial software for the last fifteen years, yet is still not used in routine well test analysis. In addition, interest in well test analysis seems to be fading, which is in sharp contrast with its popularity in 1970's, as measured by the attendance to the well test analysis sessions in the annual ATCE meetings. It is hoped that the new advances in ML and AI will reverse this trend and reinstate well test analysis as a major reservoir characterization tool.

Introduction

The objective of well test analysis (WTA) is to extract information about the well and the reservoir, from pressure and rate data measured in a producing well. Pressure is usually measured in the wellbore, as close as possible to the reservoir, whereas the rate is usually measured at the surface.

A typical test recording is pictured in Fig. 1, with producing drawdown periods alternating with build ups with zero rate, such as flow periods FP4 and FP76.

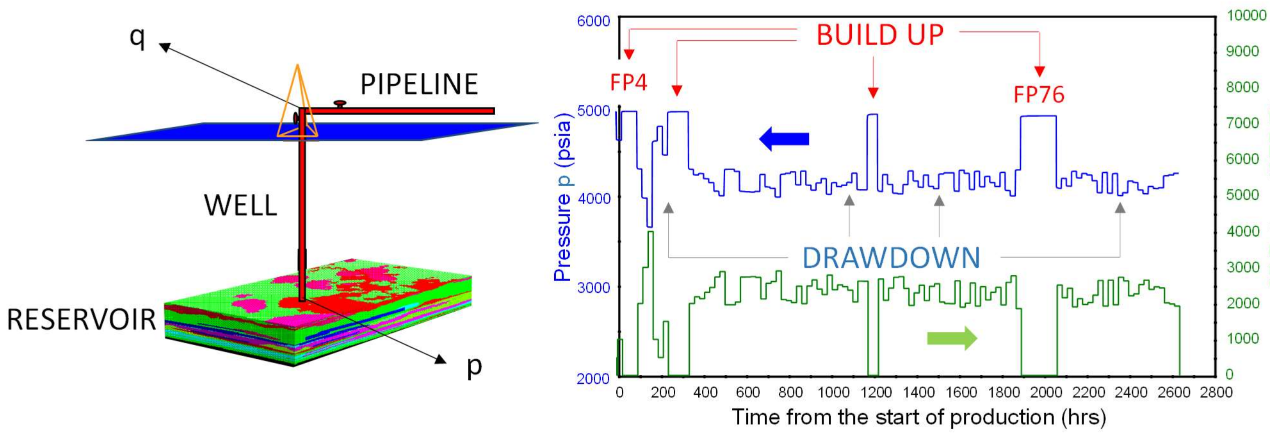

Figure 1: Example of pressure and rate recording during a well test

Over the last seventy years, the information extracted from a well test has expanded from just estimating well performance and describing the vicinity of the wellbore (nature of the fluid, permeability, well condition measured by a skin factor, average reservoir pressure), to describing the reservoir (heterogeneities, hydraulic connectivity, and distances to boundaries).

The process of interpreting well tests has also evolved, from only analyzing *radial* flow, by drawing a straight line on a plot of some function of build up pressure *vs.* some function of time, to identifying *all possible* flow regimes (linear, spherical,…). These flow regimes are then included into an *interpretation model* which must reproduce the pressure given the rate (or vice-versa), and is *consistent* with all other information available on the well and the reservoir (geology, seismic, cores, logs, completion, etc…).

Evolution of well test analysis since the 1950's

At a time when Machine Learning (ML) and Artificial Intelligence (AI) are coming-of-age and may eventually replace well test interpreters, it is worth taking stock of the progress made in well test analysis since the initial publications of the early 1950's.

This seems necessary because it has become more and more difficult for non-specialists to keep abreast of the various developments in well test analysis, due to the increase over the last decades in the number of SPE publications related to well testing. Fig. 2 shows that the total number of SPE publications as reported in OnePetro has increased from about one publication for every 50 members in the period 1963–1990, to about one publication for every 25 members after 1993[1], while the number of SPE members has increased tenfold from 15,000 in 1960 to more than 150,000 in 2020[2]. Of all the SPE papers, 20% are related to well testing and their annual number has increased from 30 in 1968 to about 400 in 2020. If one could read the annual SPE well test output in one month in 1968, it would take 13 months in 2020!

[1] The increase in the number of SPE publications is likely due to the availability of Word and PowerPoint, which have lowered significantly the cost of publishing an SPE paper, down from about 10K$ in 1980.

[2] Including students

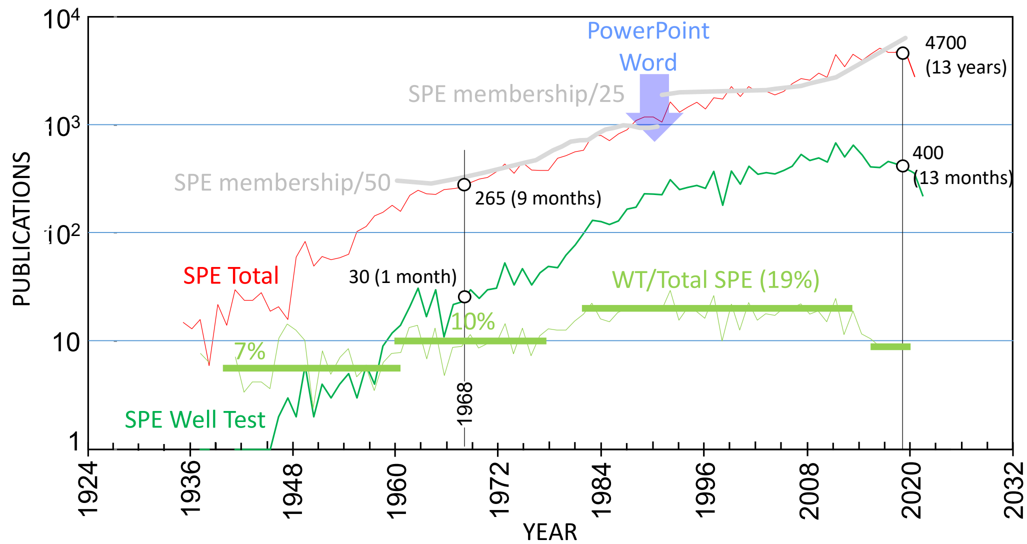

Figure 2: SPE Publications

Fig. 3 lists the major improvements that have occurred approximately 13 to 19 years apart, driven by the availability of both new types of data and new mathematical tools. Fig. 3 also indicates where the various developments took place, from the initial publications in groundwater hydrogeology[3,4].

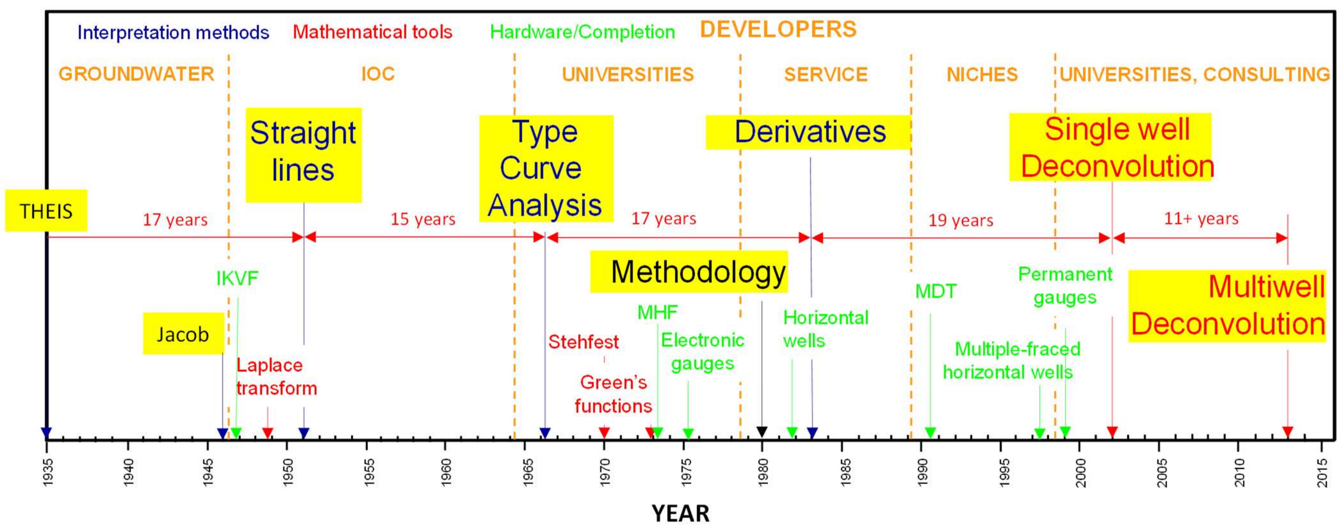

Figure 3: Evolution of well test analysis techniques

Using the computer-generated pressure and rate data set of Fig. 1, we illustrate the evolution of well test analysis over the last seventy years:

- straight-line methods of the early 1950's;
- log-log pressure plots of the late 1960's and early 1970's;
- formulation of an integrated methodology in the early 1980s;
- introduction of pressure-derivative analysis in 1983;
- derivation of a stable deconvolution algorithm in the early 2000's;

and

- its current, successful extension to multiple interfering wells.

[3] Theis, C. V.: "The Relationship between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge Using Ground-Water Storage", *Trans.*, AGU(1935) 519; *Pressure Transient Testing Methods*, Reprint Series, SPE, Richardson, Tx (1980) **14**, 27-32.

[4] Cooper, H. H. and Jacob, C. E.:" A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well-Field History," *Trans. Am. Geophys. Union* (Apr., 1946) 526-534.

Straight-line methods

The main tools for analysing well tests in the early 1950's in the oil and gas industry were the MDH[5] and the Horner[6] methods: a straight line through the pressure points is drawn where *radial flow is assumed to dominate*, on a graph of some function of pressure vs. some function of time. The straight-line slope and intercept are used to calculate permeability and skin factor. The Horner method[7] initially developed for a single drawdown followed by a build up, was extended later to multiple flow rates[8].

Multirate Horner analysis is applied in Fig. 4 to build ups FP4 and FP76 from Fig. 1. Straight-line sections seem to exist at the end of the build ups on a graph of pressure versus a superposition function, and are identified by red bands around 500 for FP4 and between 2500 and 4000 for FP76. *Assuming these correspond to radial flow*, the respective slopes and intercepts yield permeability and skin factor.

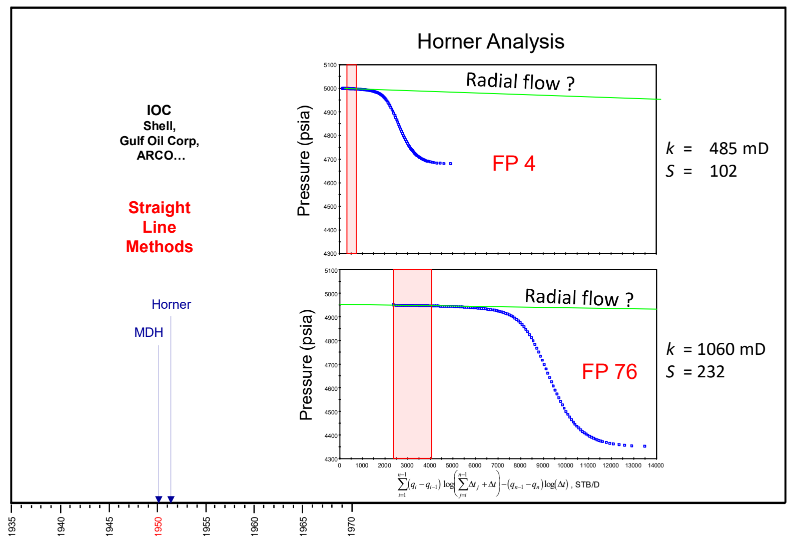

Figure 4: Straight line (Horner) analysis

However, the values calculated for permeability (485mD for FP4 and 1060mD for FP76, respectively) and for the skin factor (102 and 232) are different for the two build ups. While the skin factor may have changed over time, the permeability should not: one permeability is therefore incorrect, or even both may be incorrect.

This illustrates the main limitation of straight-line methods: there is no guarantee that a linear section identified on the pressure data does correspond to radial flow; and worse, there is no clear way to find if radial flow even exits.

Log-log pressure analysis

Log-log pressure analysis with type curves was introduced by H. J. Ramey, Jr and his PhD students in 1966[9] in an attempt to identify the "correct" radial flow straight line. It also revealed the existence of different early time behaviors, allowing the identification of an *interpretation model*.

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[5] Miller, C. C., Dyes, A. B., and Hutchinson, C. A.: "Estimation of Permeability and Reservoir Pressure from Bottom-Hole Pressure Build-up Characteristics," *Trans.*, AIME (1950) **189**, 91-104.

[6] Horner, D. R.: "Pressure Build-ups in Wells", *Proc.*, Third World Pet. Cong., E. J. Brill, Leiden (1951) **II**, 503-521. Also, *Reprint Series, No. 9 — Pressure Analysis Methods*, Society of Petroleum Engineers of AIME, Dallas (1967) 25-43.

[7] The MDH method is less reliable than the Horner method because it is affected by the duration of production before the build up being analyzed (see Gringarten, A. C., Bourdet D. P., Landel, P. A. and Kniazeff, V. J.: "A Comparison between Different Skin and Wellbore Storage Type-Curves for Early-Time Transient Analysis," *paper SPE 8205*, presented at the 54th Annual Technical Conference and Exhibition of SPE, Las Vegas, Nev., Sept. 23-26, 1979).

[8] Odeh, A. S. and Jones, L. G.: "Pressure Drawdown Analysis, Variable Rate Case," *J. Pet. Tech.* (Aug., 1965) 960.

[9] Agarwal, R.G., Al-Hussainy, R. and Ramey, H. J., Jr.: "An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow. I: Analytical Treatment," *Soc. Pet. Eng. J.* (Sept., 1970) 279.

As illustrated in Fig. 5, one plots the difference Δp between the pressure at the start of a build up, $p(\Delta t=0)$ and the pressure $p(\Delta t)$ at an elapsed time Δt in the build up, versus the elapsed time Δt, on a log-log graph. To compare several build ups, one normalizes the Y axis by plotting $\Delta p/q$, where q is the rate of the drawdown preceding the build up.

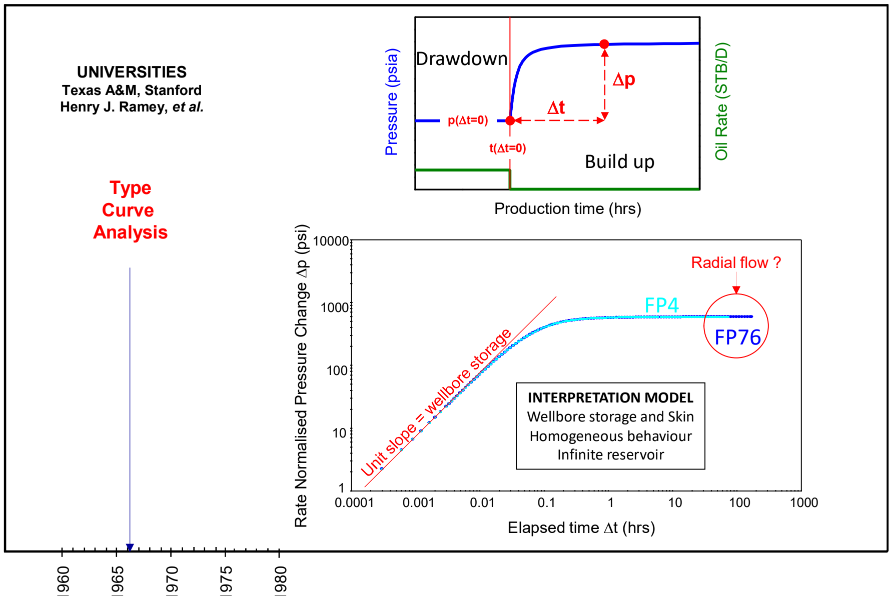

Figure 5: Log-log pressure analysis

A normalized log-log plot is shown in Fig. 5 for FP4 and FP76. The two build ups are on top of each other, which indicates that they exhibit the same behavior. The early time pressure points are aligned on a straight line of unit slope, characteristic of wellbore storage, whereas the pressure at the end *suggests* radial flow. In the absence of obvious outer boundaries, the interpretation model is therefore "a well with wellbore storage and skin in an infinite reservoir with homogeneous behavior[10]".

Quantitative analysis is performed with type curves representing the model. At the time, the published type curves for wellbore storage and skin were the Ramey[11] and the McKinley[12] type curves (Fig. 6).

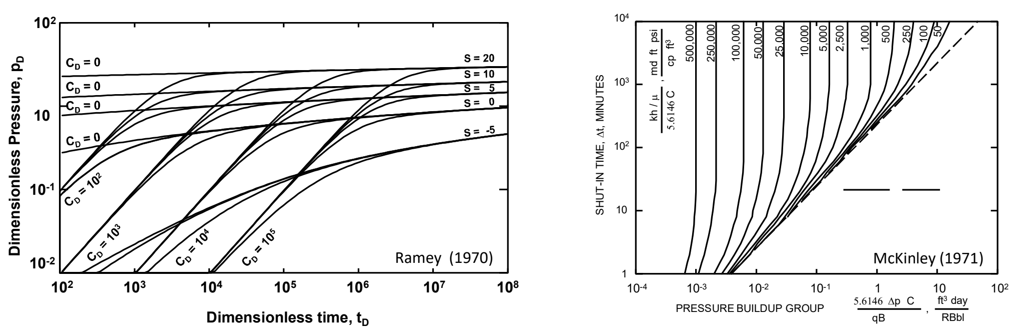

Figure 6: Wellbore storage and skin type curves

Although they all correspond to the same interpretation model, i.e. a well with wellbore storage and skin in an infinite reservoir with homogeneous behavior, they actually gave different analysis results (which at the time strongly affected the confidence in type curve analysis). This is due to parameters used in the

[10] "*Homogeneous behavior*" means that there is no variation of mobility-thickness (kh/μ) and of storativity ($\phi c_t h$) in the reservoir.
[11] Ramey, H. J., Jr.: "Short-Time Well Test Data Interpretation in The Presence of Skin Effect and Wellbore Storage," *J. Pet. Tech.* (Jan., 1970) 97
[12] McKinley, R. M.: "Wellbore Transmissibility from Afterflow Dominated Pressure Build-up Data," *J. Pet. Tech.* (July, 1971) 863.

Ramey and McKinley type curves not being independent, whereas independent parameters[13,14] are used in the two type curves shown in Fig. 7.

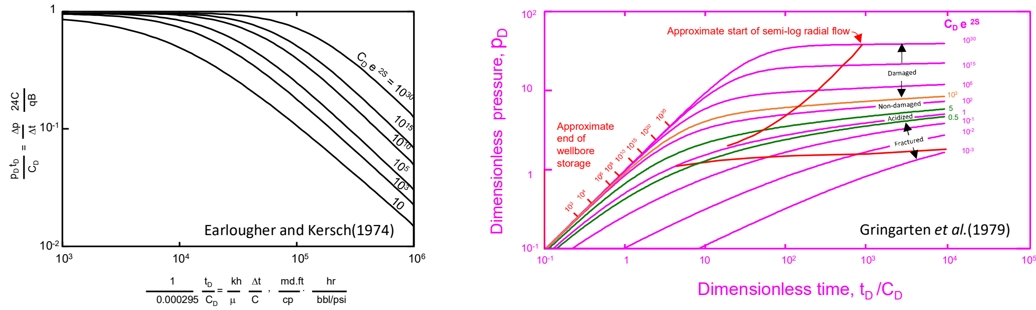

Figure 7: Wellbore storage and skin type curves using independent dimensionless parameters

The type curves in Fig. 7 are identical, except that the one on the LHS is rotated compared to the one on the RHS, and only represents non-damaged and damaged wells. The RHS type curve includes additional information related to the end of wellbore storage and the start of radial flow, and to the condition of the well, to facilitate the analysis. The type curves of Fig. 6 become identical to the type curves of Fig. 7 when redefined with the parameters of Fig. 7[15].

FP4 and FP76 are matched in Fig. 8 with the RHS type curve of Fig. 7. This time, FP4 and FP76 yield the same permeability (175mD) and the same skin factor (32). But these values are different from the ones obtained from Horner analysis, although the match appears to confirm that the late time data in both build ups are in radial flow.

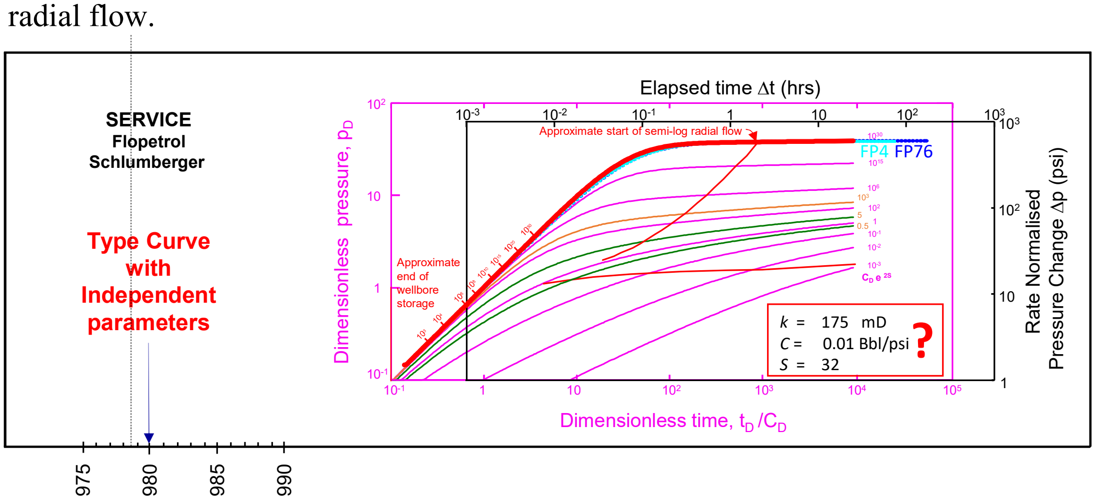

Figure 8: Log-log pressure analysis with type curve of Reference 13

Hence, it is still not clear what the permeability and skin factor should be.

Integrated methodology

To help solve this problem, a methodology or workflow was developed as described in Fig. 9[16], which involves two successive steps, namely (1) the *identification* of a well test interpretation model; and (2) the *verification* that such a model can reproduce the data AND is consistent with any other information one may have on the well and the reservoir (geology, seismic, cores, logs, completion, etc…).

[13] Gringarten, A. C., Bourdet D. P., Landel, P. A. and Kniazeff, V. J.: "A Comparison between Different Skin and Wellbore Storage Type-Curves for Early-Time Transient Analysis," *paper SPE 8205*, presented at the 54th Annual Technical Conference and Exhibition of SPE, Las Vegas, Nev., Sept. 23-26, 1979.
[14] Earlougher, R.C., Jr. and Kersch, K.M. : "Analysis of Short-Time Transient Test Data by Type Curve Matching," J. Pet. Tech. (July1974)793.
[15] The McKinley type curves differ at late times as they assume a constant pressure outer boundary.
[16] Gringarten A. C.:"From Straight lines to Deconvolution: the Evolution of the State of the art in Well Test Analysis," paper SPE 102079, presented at the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 24–27 September 2006; SPEREE (Feb. 2008) 11-1 pp. 41-62

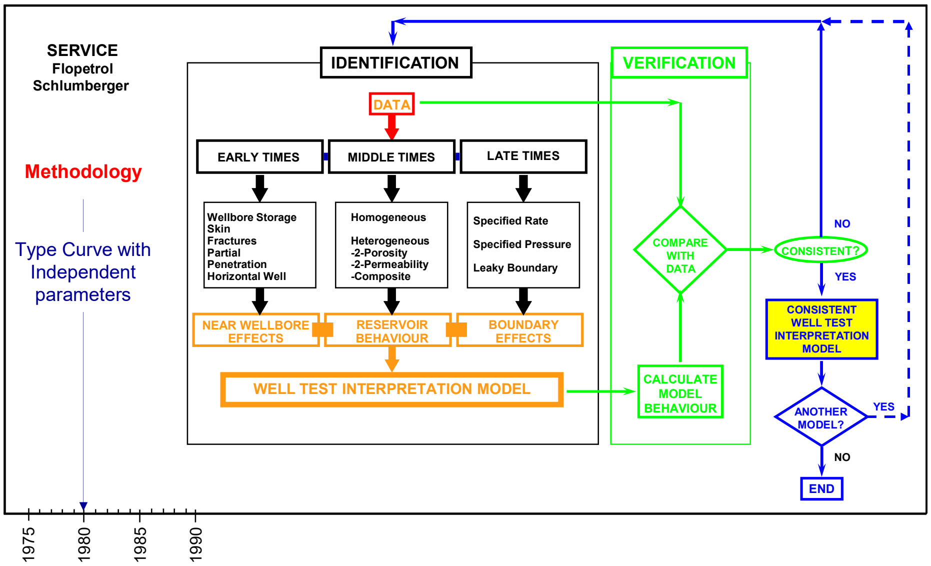

Figure 9: Well test interpretation methodology

The interpretation model is obtained from the observed well pressure by identifying and combining flow regimes (linear, bilinear, spherical, radial, etc.), which dominate at early, middle and late times[17] and yield distinctly different transient pressure behaviors. The interpretation model determines the number and the meaning of the parameters that can be obtained from the analysis.

The behavior of the diagnosed interpretation model is then compared with the actual data, and consistency checks are made between all characteristics implied by the model and the corresponding known information on the actual reservoir and measured data. If the model matches the measured data AND satisfies all the checks, it is deemed to be "*consistent*" and to represent a valid solution to the problem. If the model fails one single check, it is considered invalid. The interpretation process must be repeated to identify all possible consistent models, which can be ranked in terms of decreasing probability.

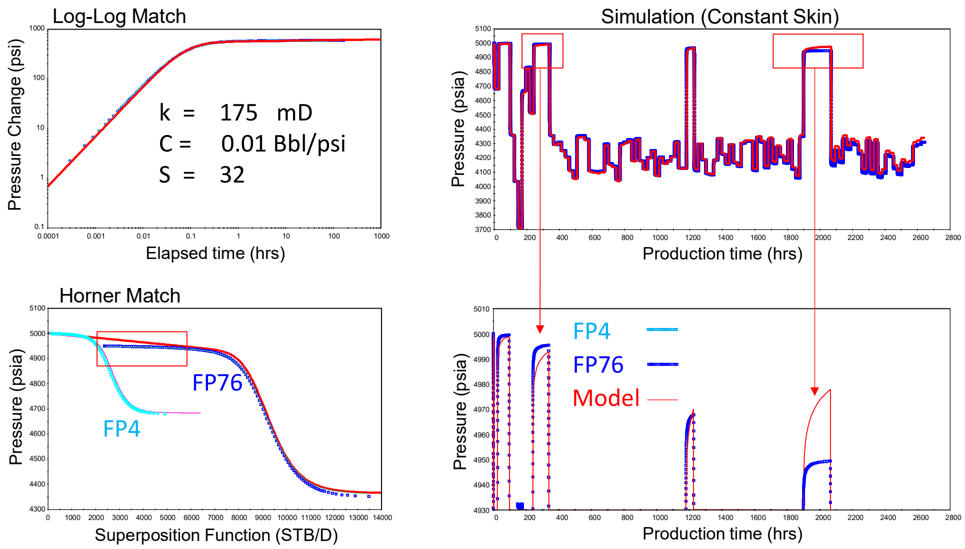

Figure 10: Verification of the log-log analysis from Fig. 8

[17] It has been observed that, although reservoirs are different in terms of physical description (type of rock, depth, pressure, size, type of fluid, fluid content, etc.), the number of possible dynamic behaviors of these reservoirs during a well test was limited to five near-wellbore effects, two basic reservoir behaviors, and three types of outer boundary effects. However, their combination can yield several thousand different interpretation models to match all observed well behaviors.

Fig. 10 illustrates the verification of the model identified in Fig. 5, namely "a well with wellbore storage and skin in an infinite reservoir with homogeneous behavior". There is an apparently good match between interpretation model and pressure date on a log-log graph (upper LHS), but there is no match on a Horner (superposition) plot (lower LHS), and there is no match on the build ups when the entire test is simulated[18] (RHS of Fig. 10).

Hence, the model "a well with wellbore storage and skin in an infinite reservoir with homogeneous behavior" is incorrect.

Pressure-derivative analysis

A major milestone in well test analysis was reached in 1983 with the introduction of the pressure derivative analysis by Bourdet *et al.* [19]: plotting, on a log-log graph, the derivative of the pressure with respect to the decimal log of the time, $d(\Delta p)/d(\log\Delta t)$, *vs.* the elapsed time Δt in a given drawdown or build up, revealed flow regimes that were difficult or impossible to identify with previous methods.

Pressure derivative analysis is applied in Fig. 11 to the build ups FP4 and FP76 from Fig. 1. The log-log plot at the top of Fig. 11 compares the FP4 and FP76 derivatives with the model for "a well with wellbore storage and skin in an infinite reservoir with homogeneous behavior" obtained from the log-log pressure analysis in Fig. 8. Whereas there is an apparent good match on the pressure, it is clear that the model does not match the derivative: contrary to what was diagnosed in Fig. 8, there is no radial flow, which would appear as a horizontal line on the derivative.

The log-log plot of the derivative *reveals* other flow regimes, however, namely *limited entry* (straight line with negative half-unit slope at middle times); and either a constant pressure outer boundary or a closed reservoir (as FP76 is a build up) or possible derivative end effects[20], *suggested* by the derivative decreasing to zero at late times. These flow regimes could not be identified on the log-log plot of the pressure.

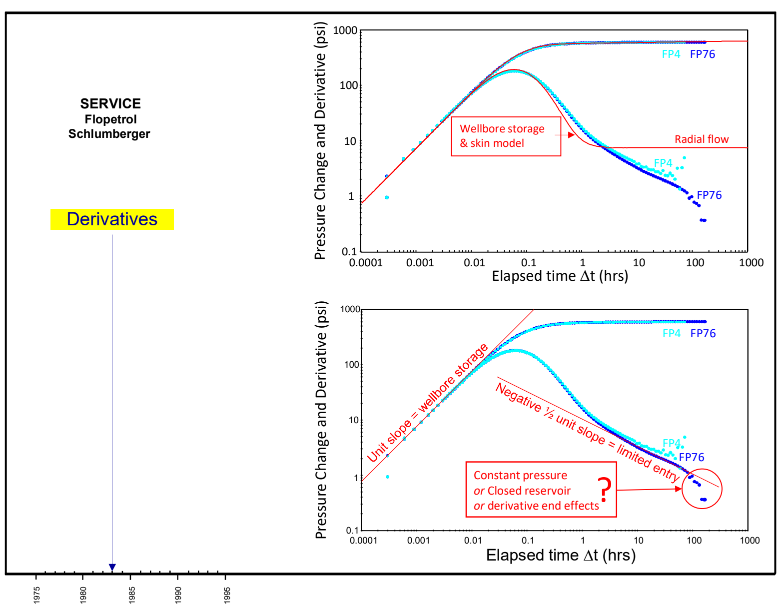

Figure 11: Pressure derivative analysis

[18] Matching of the drawdowns in the simulation of the pressure history can be improved by adjusting the skin factor whereas the build ups must match otherwise the interpretation model is incorrect.

[19] Bourdet, D. P., Whittle, T. M., Douglas, A. A. and Pirard, Y. M.: "A New Set of Type Curves Simplifies Well Test Analysis," *World Oil* (May, 1983) 95-106.

[20] Gringarten A. C.:"From Straight lines to Deconvolution: the Evolution of the State of the art in Well Test Analysis," paper SPE 102079, presented at the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 24–27 September 2006; SPEREE (Feb. 2008) 11-1 pp. 41-62

Assuming a closed boundary, the model diagnosed with the pressure derivative is “a well with wellbore storage and skin and limited entry in a closed reservoir with homogeneous behavior”. This model is validated by good matches with actual data, on log-log, Horner and pressure history plots in Fig. 12. On the other hand, validation failed for both a constant pressure outer boundary or derivative end effects.

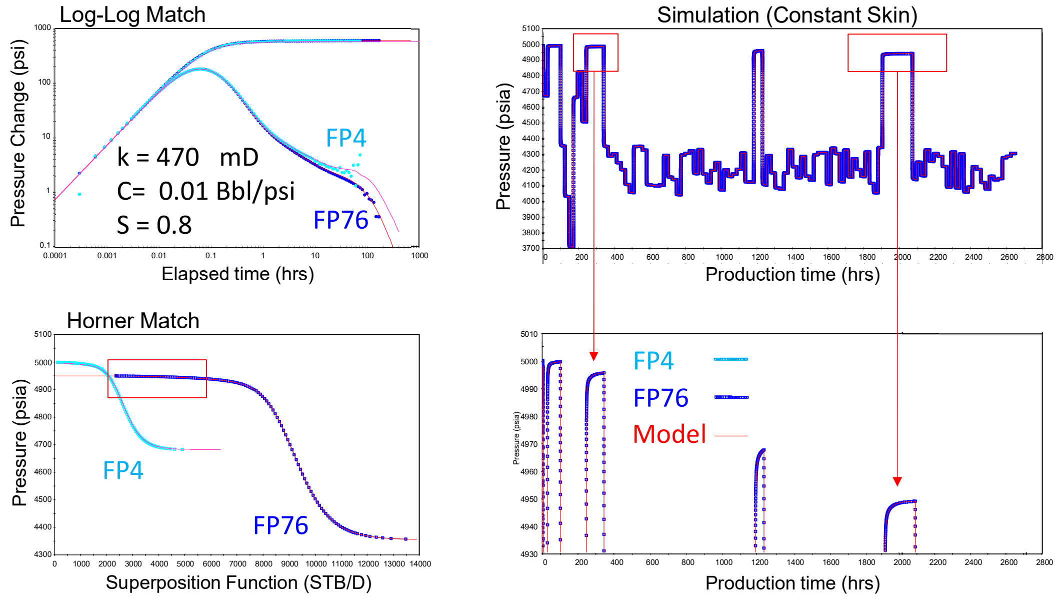

Figure 12: Verification of the pressure derivative analysis in Fig. 10

The complete analysis results are listed in Table 1. As the test is computer-generated, we can compare the analysis results with their design values. The agreement is very good. In practice, because of errors in pressure and rate measurements and uncertainties in the values of the well, fluid and reservoir parameters used to calculate the test analysis results, interpretation model parameters cannot be estimated with a better accuracy than that indicated in the RHS column of Table 1[21].

MODEL	a well with wellbore storage and skin and limited entry in a closed homogeneous reservoir					
Parameter		**Analysis**		**Design**	**Difference**	**Typical field uncertainty**
Initial pressure	(pav)i	5000	psia	5000	0%	± 5 psia
Horizontal permeability	k(xy)	470	mD	500	-6%	± 20%
Vertical permeability	k(z)	0.54	mD	0.5	8%	± 20%
Wellbore storage coefficient	C	0.01	bbl/psi	0.01	0%	± 20%
Penetration ratio	hw/h	0.06		0.05	20%	± 20%
Wellbore skin effect	S(w)	0.8		0	+ 0.8	± 0.5
Reservoir area	A	$6\ 10^7$	ft^2	$6\ 10^7$	0%	± 30%

Table 1: Results for the pressure derivative analysis and uncertainties on these results

Single well deconvolution

The interpretation model of Fig. 11 was obtained by *assuming* a closed outer boundary, which the verification in Fig. 12 proved correct. One way to remove the need to make such an assumption is to use deconvolution[22].

[21] Azi, A. C., Gbo, A., Whittle, T and Gringarten, A.C.:”Evaluation of confidence intervals in well test interpretation results,” paper SPE 113888 presented at the 2008 SPE Europec/EAGE Annual Conference and Exhibition, Italy, 9–12 June 2008

[22] von Schroeter, T., Hollaender, F., Gringarten, A. C.:”Analysis of Well Test Data From Permanent Downhole Gauges by Deconvolution," paper SPE 77688, presented at the 2002 SPE Annual Technical Conference and Exhibition, San Antonio, TX, Sept. 29 – Oct. 2, 2002; *Society of Petroleum Engineers Journal*, (Dec. 2004) 375-390

Deconvolution is actually not an interpretation technique, but is data processing with an algorithm: it converts pressures at variable rates into a single drawdown at constant rate, with a duration equal to the duration of the test. This is illustrated in Fig. 13: the 2644-hour pressure history is converted into a single 2644-hour constant rate drawdown, which obviously must contain more late-time information than the longest build up period analyzable by previous methods, i.e. the 168-hour build up FP76.

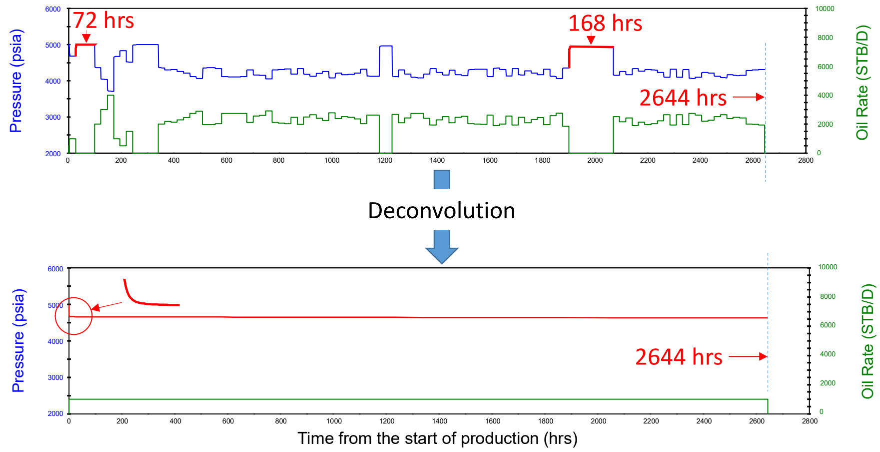

Figure 13: Conversion of a pressure history to a single drawdown by deconvolution

Deconvolution, applied to the entire pressure in Fig. 14, clearly *shows* a unit slope straight line at late times, which is characteristic of a closed reservoir. This unit-slope straight line is reached after 300 hours, i.e. beyond the duration of FP76, and lasts until the end of the deconvolved pressure (2644 hours), a gain of over one log cycle compared to the duration of FP76. The longer the rate history, the greater the gain. In addition, deconvolution can correct erroneous rates and even recreate an entire rate history[23].

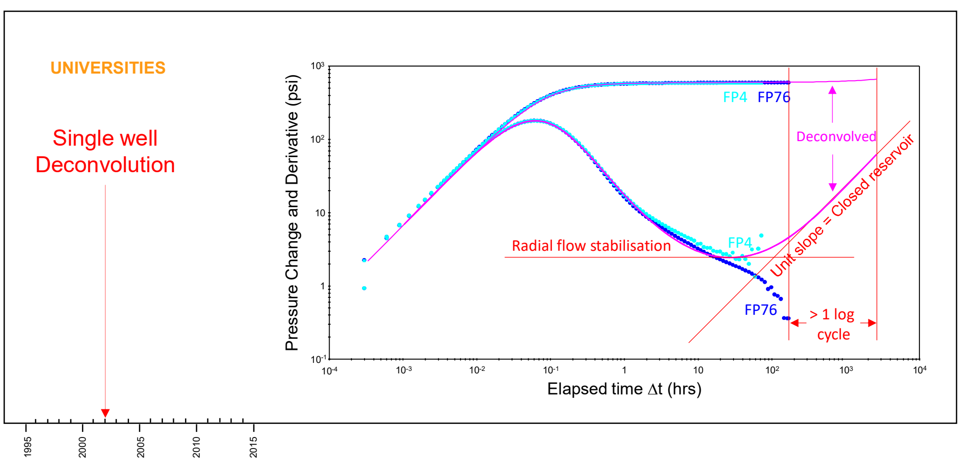

Figure 14: Deconvolution

Multiwell deconvolution

The single well deconvolution algorithm has been extended[24] to any number of interfering wells[25] in a reservoir. Multiwell deconvolution not only extracts the pressure responses to production for *every* well in the reservoir, but also captures and separate the *interference effects* of production from any one well to any other.

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[23] Aluko, O.A., Cumming, J., and Gringarten, A.C.:"Using Deconvolution to Estimate Unknown Well Production from Scarce Wellhead Pressure Data", SPE paper SPE-201667-MS presented at the SPE Annual Technical Conference and Exhibition held in Houston, Texas, 11 - 14 Oct 2020.

[24] Levitan, M. M.: "Deconvolution of Multiwell Test Data", paper SPE102484 presented at the SPE Annual Technical Conference and Exhibition held in San Antonio, Texas, 24—27 September 2006; *Society of Petroleum Engineers Journal*, **12** (4): 420–428

[25] Cumming, J. A., Wooff, D. A., Whittle, T. M. and Gringarten, A. C.:"Multiple Well Deconvolution," paper SPE 166458 presented at the SPE Annual Technical Conference and Exhibition held in New Orleans, Louisiana, USA, 30 September–2 October 2013.

Fig. 15 show the results of multiwell deconvolution applied to six interfering producers and two injectors in a North Sea oil reservoir[26]. Well locations are indicated at the top LHS, with faults from seismic interpretation pictured in brown. Although multiwell deconvolution processes all the wells together, only results for well P13 are shown in Fig. 15. The log-log plot at the lower RHS of Fig. 15 compares the longest actual build up with single well (in green) and multiwell (in red) deconvolution results. While the single well deconvolved derivative clearly shows interferences from the other wells after 100 hours, these are removed by multiwell deconvolution[27].

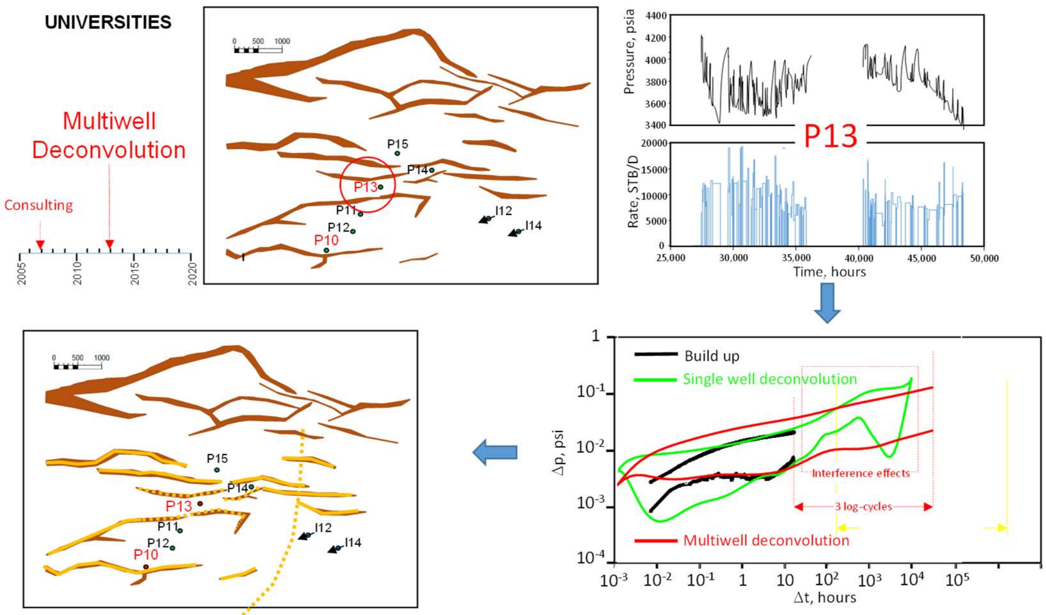

Figure 15: Example of multiwell deconvolution

The multiwell deconvolved derivatives and interferences effects were analyzed for permeability and boundary effects. Results are shown in the lower LHS of Fig. 15 where solid and broken yellow lines represent sealing and semi-sealing faults, respectively. Some faults had to be moved and their sealing factor changed, and extra faults had to be added to obtain a good match (for instance, the faults around P13 had to be narrowed and made slightly non-sealing to create a tighter channel structure).

Interferences between wells also provide additional information on the reservoir. For instance, the interference response between well P13 and well P10 suggested a geological change between the two wells, thus confirming available geological knowledge of the field (certain sands vary over the field and pinch-outs occur leading to changes in porosity and storativity). Interference also provides the storativity ($\phi c_t h$) between P13 and P10 (1.8 x 10^{-4}ft/psi *vs*. an estimate of 1.7 x 10^{-4} ft/psi from known reservoir values).

Discussion and conclusions

The evolution of well test analysis over the last seventy years is summarized in Fig. 16. Well test analysis has evolved from being a tool for evaluating well performance to being able to fully characterize a reservoir. As a result, one can extract much more information and with much higher confidence from well test analysis than ever before. While it is difficult to predict future developments, it is clear that these will have to provide significant further improvements in both the identification and the verification of the interpretation model.

[26] Thornton E., Cumming, J., Mazloom J, and Gringarten, A.C.:"Application of Multiple Well Deconvolution Method in a North Sea Field," Paper SPE 174353 presented at the SPE Europec/EAGE Annual Conference, Madrid, Spain, 1-4 June 2015.

[27] The skin factor and wellbore storage coefficient of the deconvolved derivative is an average from different wells and different flow periods at early times and is not expected to match those of a particular build up.

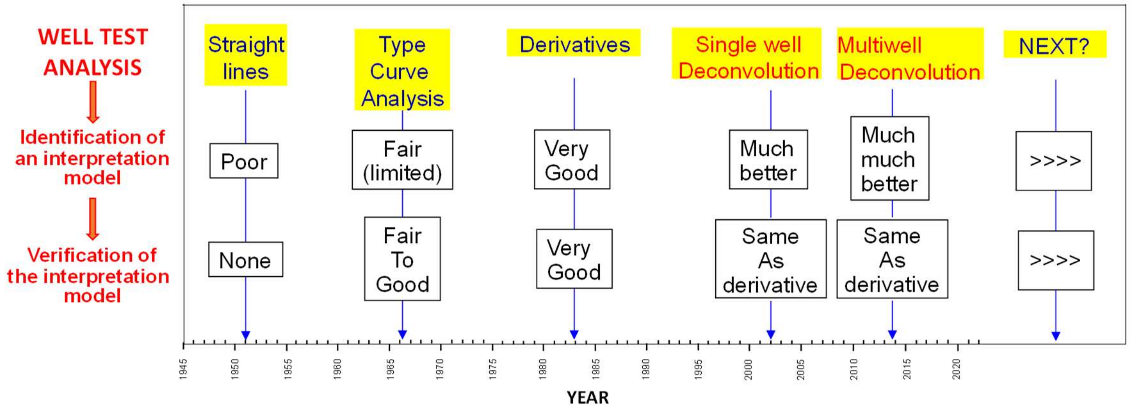

Figure 16: Evolution of well test interpretation methods

On the other hand, the process has become much more complex, and requires the use of dedicated software. But although the development of commercial software has been a factor in making practicing engineers aware of new developments in well test analysis, acceptance has been slow: for instance, single well deconvolution is still not used routinely in well test analysis, although a stable algorithm was developed twenty years ago and has been included in commercial software for the last fifteen years. Multiwell deconvolution, still being developed, is even further from routine use.

Interest in well test analysis seems to be fading, which is in sharp contrast with its popularity in the 1970's, as measured by the attendance to the well test analysis sessions in the SPE Annual Technical Conference and Exhibition meetings. One reason may be the increasing complexity mentioned above. Another reason is that WTA is often taught, wrongly, as "basic" and "advanced" which is mistaken for "practical" (for everybody) and "esoteric" (for experts only). Other factors include regular well testing being no longer mandatory in many oil provinces; well testing (and data acquisition in general) being considered an unnecessary cost in unconventionals; and dedicated WTA teams having disappeared in many oil companies.

Well test analysis is a discipline in its own right, quite different but complementary to reservoir simulation. It is hoped that the new advances in ML and AI, by making well test analysis easier, will help reinstate it as a major reservoir characterization tool.

Nomenclature

C	Wellbore storage coefficient, Bbl/D	$\phi c_t h$	storativity, ft/psi
c_t	total compressibility, psi^{-1}	ϕ	porosity, %
$d(\Delta p)/d(\log\Delta t)$	pressure derivative with respect to log of time, psi	μ	fluid viscosity, cp
k	permeability, mD	AI	Artificial Intelligence
h	reservoir thickness, ft	IKVF	Infinite conductivity vertical fracture
$p(\Delta t=0)$	pressure at the start of a flow period	IOC	International oil company
$p(\Delta t)$	pressure at Δt in a flow period	LHS	Left Hand Side
q	flow rate, Bbl/D	MDH	Miller, Dyes and Hutchinson
r_w	well radius, ft	MDT[28]	Modular formation dynamics tester
S	skin factor	ML	Machine Learning
Δt	elapsed time in a flow period	RHS	Right Hand Side
kh/μ	fluid mobility-thickness, mD.ft/cp	WTA	Well Test Analysis

$$p_D = \frac{kh}{141.2\Delta q B\mu}\Delta p \qquad \frac{t_D}{C_D} = 0.000295\frac{kh}{\mu C}\Delta t \qquad C_D e^{2S} = \frac{0.8936}{\phi c_t h r_w^2} C e^{2S}$$

[28] Mark of Schlumberger