

STRAIGHT LINE METHODS

NEAR WELLBORE EFFECTS

- Wellbore Storage
- High Conductivity Fracture
- Low Conductivity Fracture
- Limited Entry

Δp vs Δt

Δp vs $(\Delta t)^{1/2}$

Δp vs $(\Delta t)^{1/4}$

Δp vs $(\Delta t)^{-1/2}$

RESERVOIR BEHAVIOUR

- Homogeneous Behaviour
- Double porosity Behaviour

Δp vs $\log \Delta t$

1 line

2 parallel lines

BOUNDARY EFFECTS

- Sealing Fault
- Channel
- Two perpendicular Faults
- Closed reservoir

Δp vs $\log \Delta t$ (double slope)

Δp vs $(\Delta t)^{1/2}$

Δp vs $\log \Delta t$ (quadruple slope)

Δp vs Δt (Drawdown only)

T. J. Nowak, Union Oil Co. of California, Santa Fe Springs, Calif.)

Discussion of “Analysis of pressure build up curves”, Perrine, Dril. and Prod. Practice, 1 Jan 1956)

In California, the methods have not been widely accepted for field application. Probably two reasons are responsible for their lack of popularity.

The first reason is that a number of variations have been reported in the analytical techniques. Differences in nomenclature, in treatment, and in the claims made for the various methods have led to some confusion in their application.

The Muscat method

¹Muskat, Morris: Use of Data on the Build-up of Bottom-hole Pressures,” *Trans. Am. Inst. Mining Met. Engrs. (Petroleum Development and Technology)*, **123**, 44 (1937).

The Miller, Dyes, and Hutchinson Method

²Miller, C. C; Dyes, A. B; and Hutchinson, C. A., Jr: The Estimation of Permeability and Reservoir Pressure from Bottom-hole Pressure Build-up Characteristics, *Trans. Am. Inst. Mining Met. Engrs. (Petroleum Development and Technology)*, **189**, 91 (1950).

The Horner Method

⁵Horner, D. R: Pressure Build-up in Wells, *Proc. Third World Pet. Congress (II)*, 503, The Hague (1951).

The Thomas Method

⁶Thomas, G. B: Analysis of Pressure Build-up Data, *Trans. Am. Inst. Mining Met. Engrs. (Petroleum Development and Technology)*, **198**, 125 (1953).

The Van Everdingen Method

⁷Van Everdingen, A. F: The Skin Effect and its Influence on the Productive Capacity of a Well, *Trans. Am. Inst. Mining Met. Engrs. (Petroleum Development and Technology)*, **198**, 171 (1953).

The Hurst Method

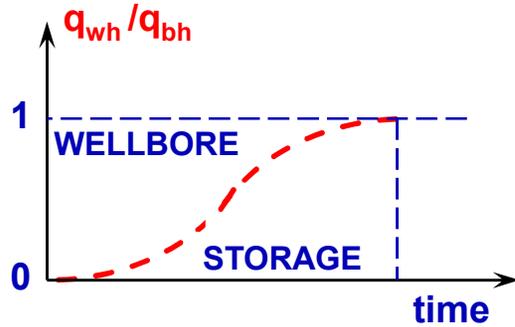
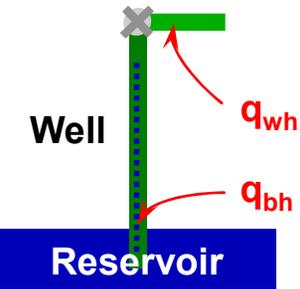
⁸Hurst, William: Establishment of the Skin Effect and its Impediment to Fluid Flow into a Well Bore, *Pet. Engr.* **25** [11] B6 (1953).

The Arps Method

⁹Arps, J. J: How Well Completion Damage can be Determined Graphically, *World Oil*, **140** [5] 225 (1955).

The second reason is that all the methods are based on stringently imposed well-fluid-reservoir assumptions. It is believed by many engineers that in practice the physical conditions of the production system depart sufficiently from the assumptions to vitiate the results of any analysis.

STRAIGHT LINE METHODS FOR WELLBORE STORAGE (Early Times)

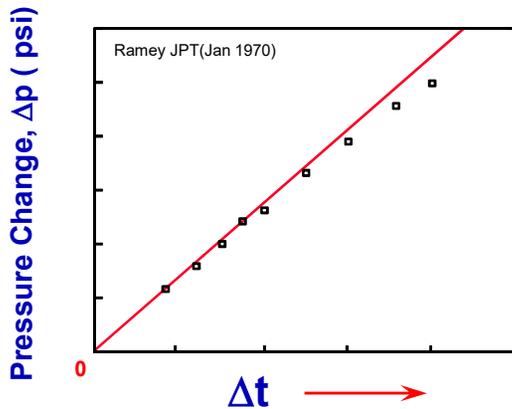


Specialised Plot

$$\Delta p = \frac{\Delta q B}{24C} \Delta t$$

Horner Plot

NONE



$$m_{WB} = \frac{\Delta q B}{24C} \quad \text{psi/hr} \qquad C = \frac{\Delta q B}{24m_{WB}} \quad \text{Bbl/psi}$$

Well full

Liquid level in well

$$C, \text{ Bbl/psi} = c_{\text{well}} V_{\text{well}}$$

$$C, \text{ Bbl/psi} = V_u / [(\rho/144)g/g_c]$$

Oil, surface shut-in= 0.01

Gas, surface shut-in= 0.05

Oil= 0.05

Oil, downhole shut-in= 0.0001

Wireline formation tester= 10⁻⁹

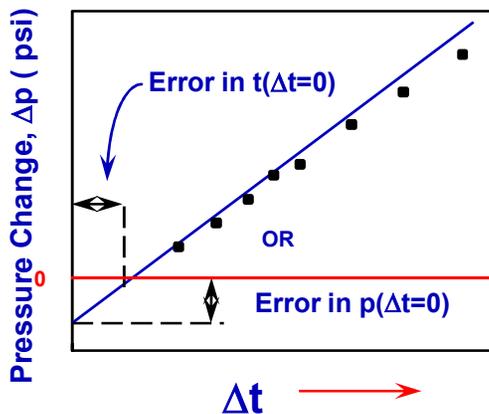
V_w is the well volume in Bbl and V_u the well volume per unit length in Bbl/ft

c_w is the average compressibility of wellbore fluids

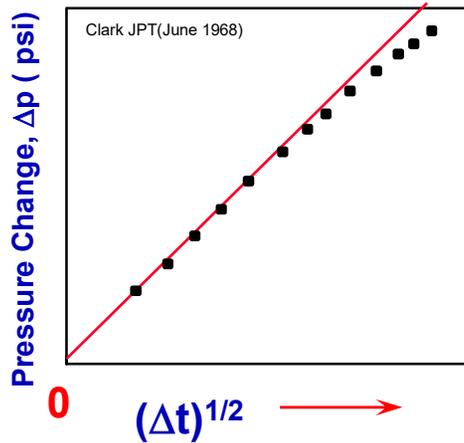
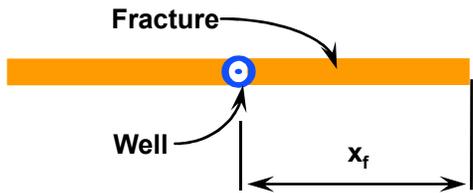
ρ is the average specific gravity of wellbore fluids in lbf/cuft

g_c is a units conversion factor to convert units of mLt^{-2} to the desired force unit (=32.1740 lbf.ft/s².lbf)

g [Lt^{-2}] is the acceleration (=32.1740 ft/s²) $g/g_c=1$

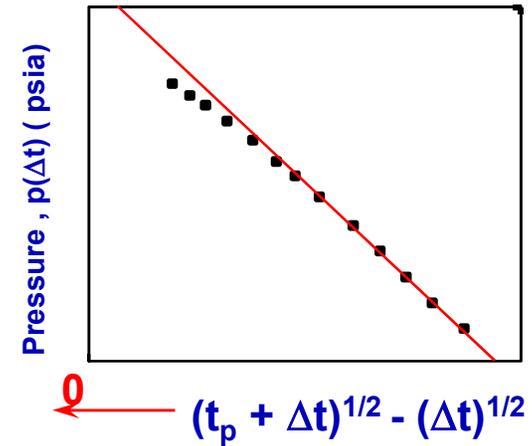


STRAIGHT LINE METHODS FOR A HIGH CONDUCTIVITY FRACTURE (Early times)



Specialised Plot

$$\Delta p = 4.06 \frac{\Delta q B}{h x_f} \sqrt{\frac{\mu}{\phi c_t k}} \sqrt{\Delta t}$$

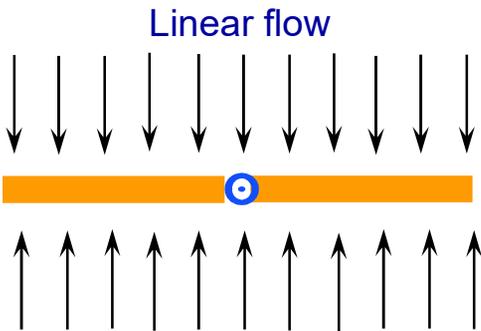


Horner Plot

$$p(\Delta t) = \bar{p}_i - 4.06 \frac{\Delta q B}{h x_f} \sqrt{\frac{\mu}{\phi c_t k}} \left[(t_p + \Delta t)^{1/2} - (\Delta t)^{1/2} \right]$$

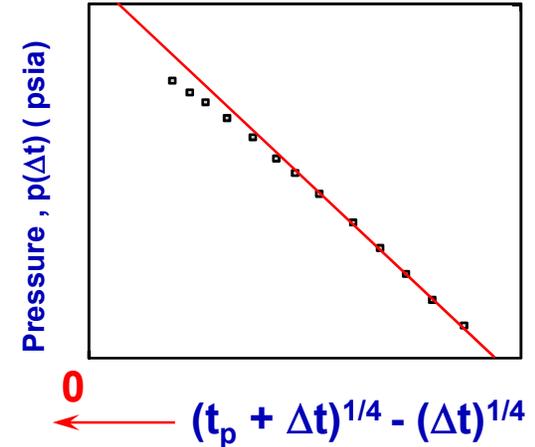
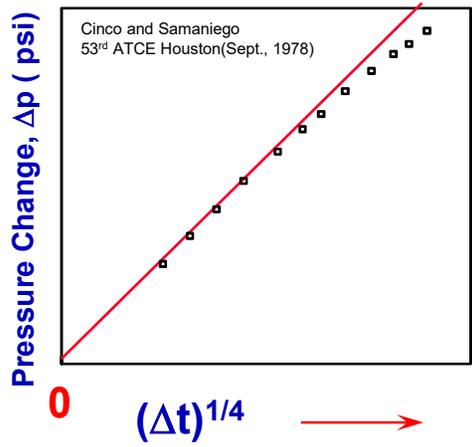
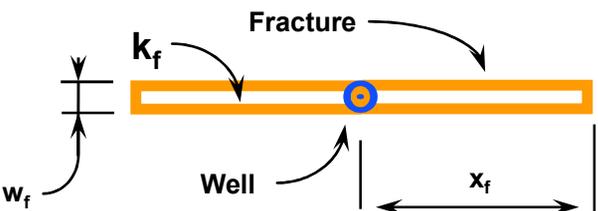
$$m_{HKF} = 4.06 \frac{\Delta q B}{h x_f} \sqrt{\frac{\mu}{\phi c_t k}} \quad \text{psi/(hr)}^{1/2}$$

$$k x_f^2 = 16.52 \frac{\mu}{\phi c_t} \left(\frac{\Delta q B}{h m_{HKF}} \right)^2 \quad \text{mD.sqft}$$



Linear flow

STRAIGHT LINE METHOD FOR A LOW CONDUCTIVITY FRACTURE (Early Times)

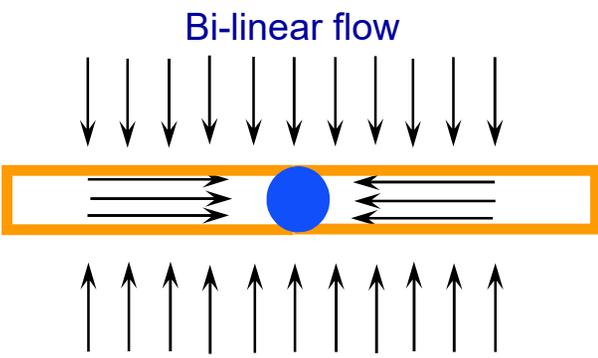


Specialised Plot

Horner Plot

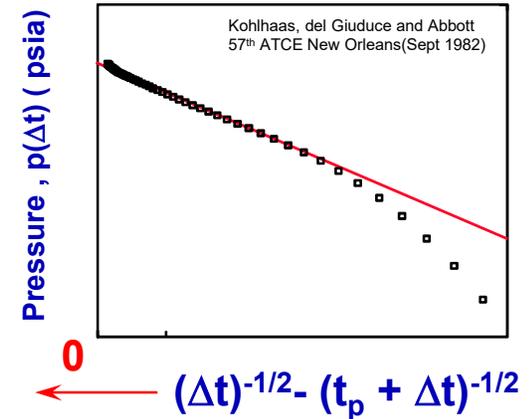
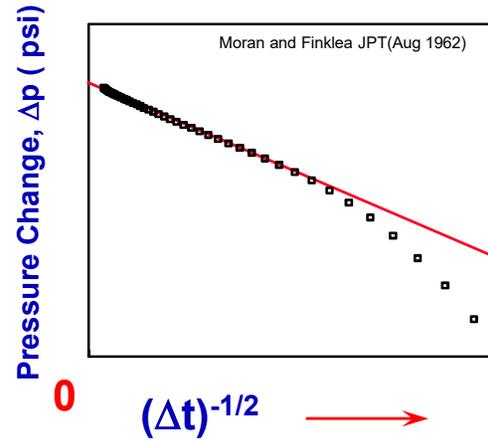
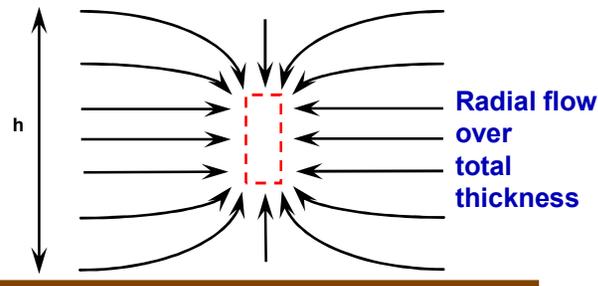
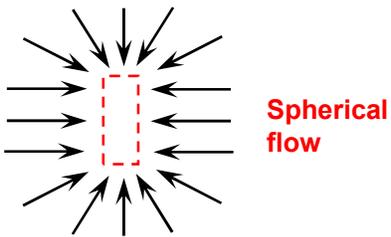
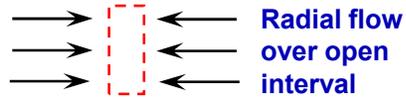
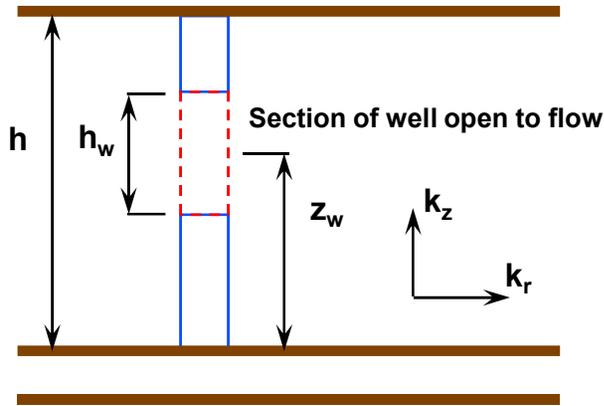
$$\Delta p = 44.1 \frac{\Delta q B \mu}{h \sqrt{k_f w_f} \sqrt[4]{\phi \mu c_t k}} \sqrt[4]{\Delta t}$$

$$p(\Delta t) = \bar{p}_i - 44.1 \frac{\Delta q B \mu}{h \sqrt{k_f w_f} \sqrt[4]{\phi \mu c_t k}} \left[(t_p + \Delta t)^{1/4} - (\Delta t)^{1/4} \right]$$



$$m_{LKF} = 44.1 \frac{\Delta q B \mu}{h \sqrt{k_f w_f} \sqrt[4]{\phi \mu c_t k}} \quad \text{psi/(hr)}^{1/4} \quad k_f w_f = 1944.8 \left(\frac{\Delta q B \mu}{h m_{LKF}} \right)^2 \frac{1}{\sqrt{\phi \mu c_t k}} \quad \text{mD.ft}$$

STRAIGHT LINE METHOD FOR SPHERICAL FLOW (Middle Times)



Specialised Plot

Horner Plot

$$\Delta p = 70.6 \frac{\Delta q B \mu}{k_{SPH} r_{SPH}} - 2452.9 \frac{\Delta q B \mu \sqrt{\phi \mu c_t}}{k_{SPH}^{3/2} \sqrt{\Delta t}}$$

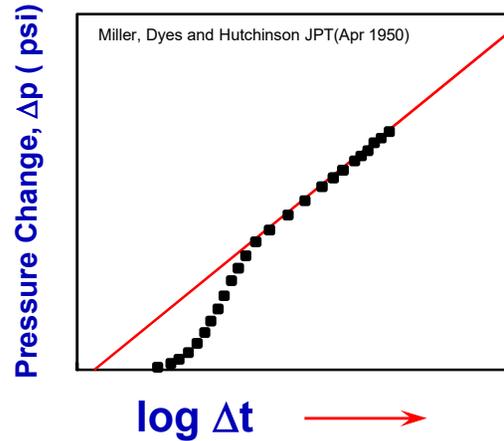
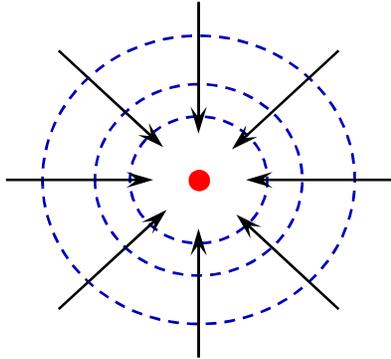
$$k_{SPH} = \sqrt[3]{k_r^2 k_z}$$

$$p(\Delta t) = \bar{p}_i - 2452.9 \frac{\Delta q B \mu (\phi \mu c_t)^{1/2}}{(k_{SPH})^{3/2}} \left[(\Delta t)^{-1/2} - (t_p + \Delta t)^{-1/2} \right]$$

r_{sp} = radius of sphere into which flow converges

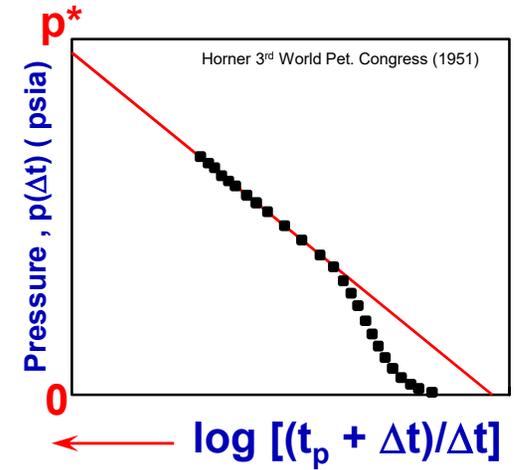
$$m_{SPH} = 2452.9 \frac{\Delta q B \mu (\phi \mu c_t)^{1/2}}{(k_{SPH})^{3/2}} \quad k_{SPH} = \left[\frac{2452.9 \Delta q B \mu (\phi \mu c_t)^{1/2}}{m_{SPH}} \right]^{2/3} \text{ mD}$$

STRAIGHT LINE METHOD FOR RADIAL FLOW (Middle Times)



Specialised Plot (MDH)

$$\Delta p = 162.6 \frac{\Delta q B \mu}{kh} \left(\log \Delta t + \log \frac{k}{\phi \mu c_i r_w^2} - 3.23 \right)$$



Horner Plot

$$p(\Delta t) = \bar{p}_i - 162.6 \frac{\Delta q B \mu}{kh} \log \frac{t_p + \Delta t}{\Delta t}$$

$$m = 162.6 \frac{\Delta q B \mu}{kh}$$

psi/(log cycle)

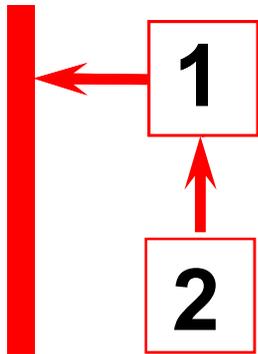
$$\frac{kh}{\mu} = 162.6 \frac{\Delta q B}{m}$$

mD.ft/cp

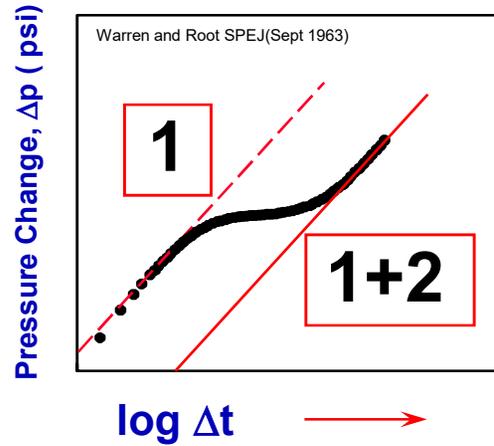
STRAIGHT LINE METHOD FOR HETEROGENEOUS BEHAVIOUR

(Middle Times)

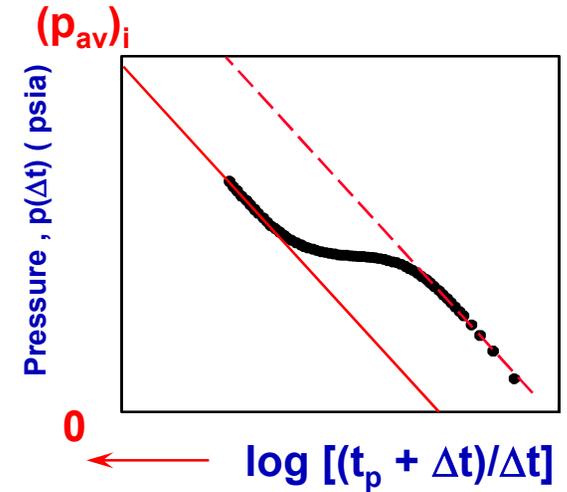
Double-porosity



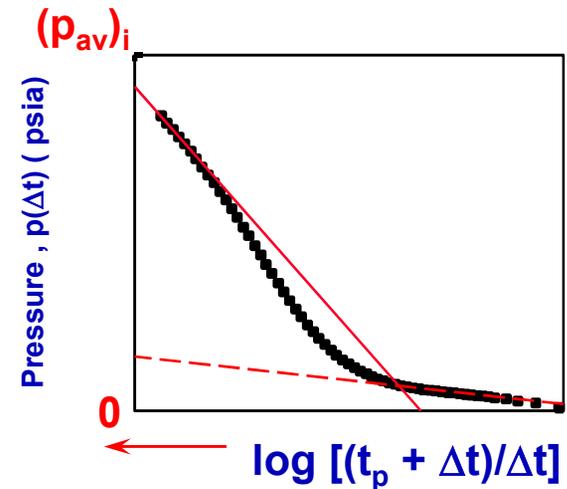
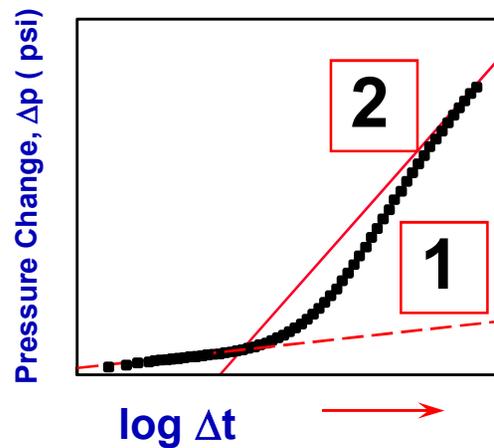
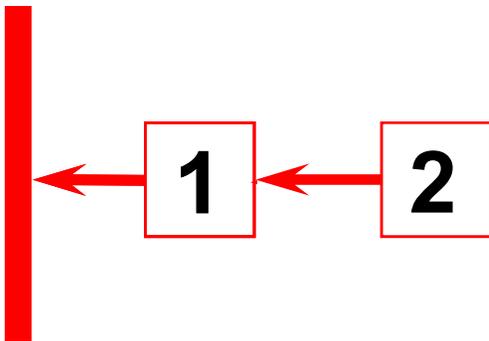
Specialised Plot (MDH)



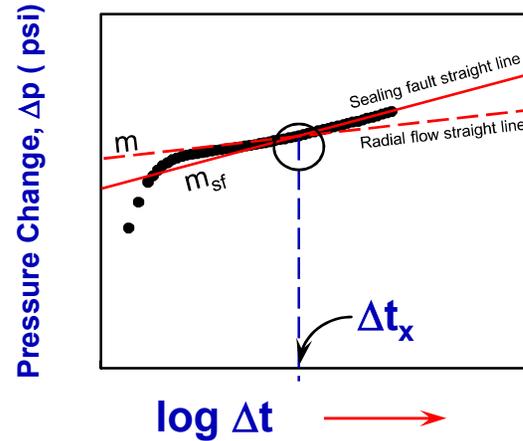
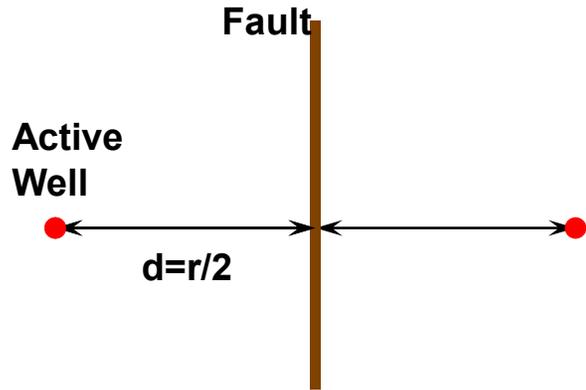
Horner Plot



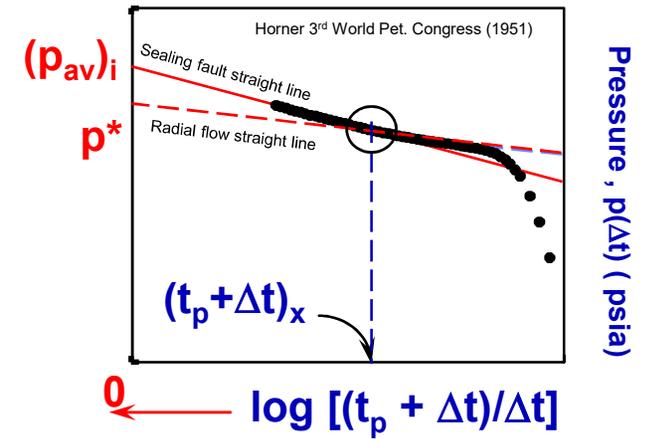
Composite



STRAIGHT LINE METHOD FOR ONE SEALING FAULT (Late Times)



Specialised Plot



Horner Plot

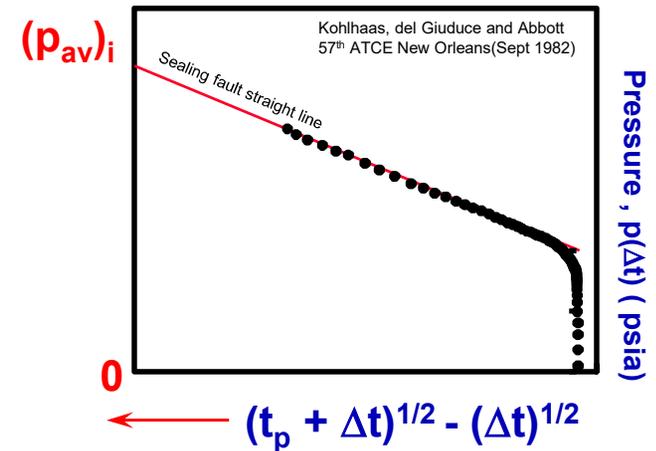
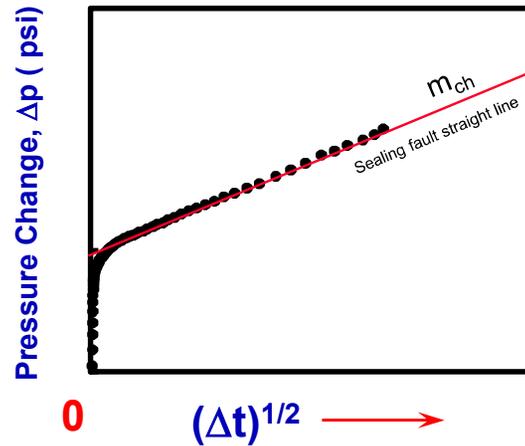
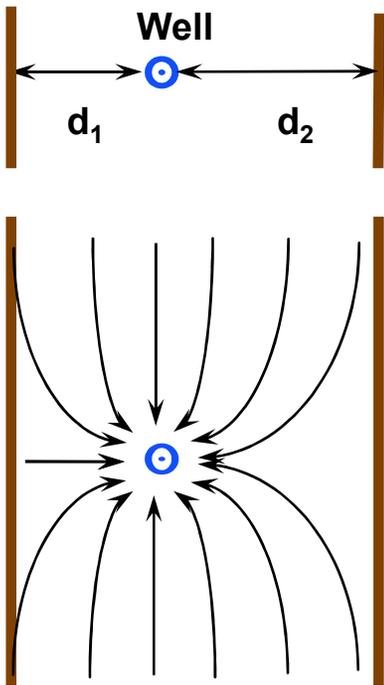
$$\Delta p = 2 \left(162.6 \frac{\Delta q B \mu}{kh} \right) \left[\log \Delta t + \log \frac{k}{\phi \mu c_t} - 3.23 + 0.43S - \log(rr_w) \right]$$

$$p(\Delta t) = \bar{p}_i - 2(162.6) \frac{\Delta q B \mu}{kh} \log \frac{t_p + \Delta t}{\Delta t}$$

$$m_{sf} = 2(162.6) \frac{\Delta q B \mu}{kh} = 2m \quad \text{psi}/(\log \text{ cycle})$$

$$d = 0.01217 \left(\frac{k \Delta t_x}{\phi \mu c_t} \right)^{1/2} \quad \text{ft}$$

STRAIGHT LINE METHOD FOR CHANNEL BOUNDARIES (Late Times)



Specialised Plot

Horner Plot

$$\Delta p = 8.133 \frac{\Delta q B}{h(d_1 + d_2)} \sqrt{\frac{\mu}{k\phi c_i}} \sqrt{\Delta t} + 141.2 \frac{\Delta q B \mu}{kh} S_{ch}$$

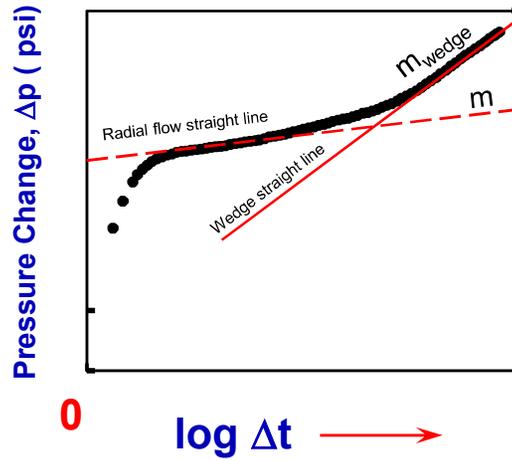
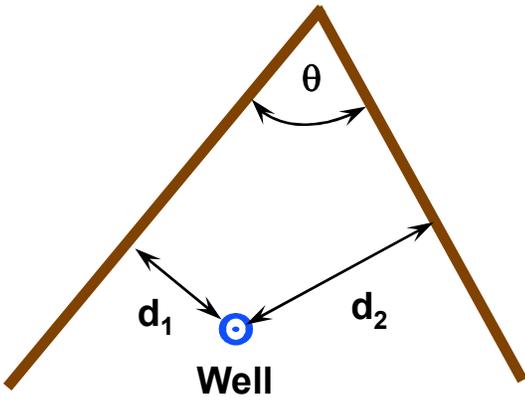
$$S_{ch} = \ln \frac{d_1 + d_2}{2\pi r_w} - \ln \left(\sin \frac{\pi d_1}{d_1 + d_2} \right)$$

$$p(\Delta t) = \bar{p}_i - 8.133 \frac{\Delta q B}{h(d_1 + d_2)} \sqrt{\frac{\mu}{k\phi c_i}} \left[(t_p + \Delta t)^{1/2} - (\Delta t)^{1/2} \right]$$

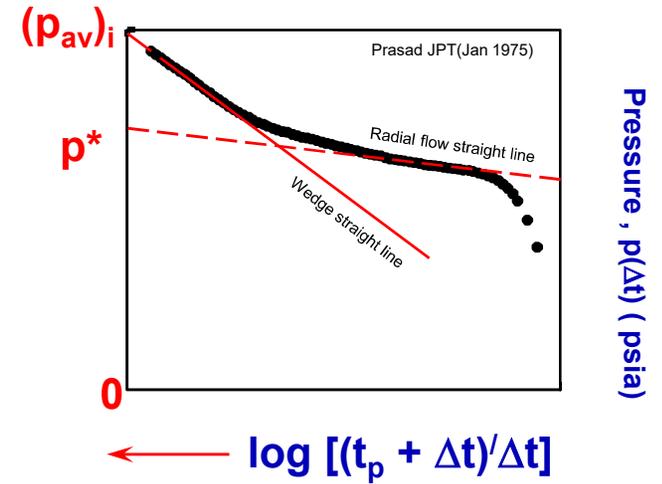
$$m_{ch} = 8.133 \frac{\Delta q B}{h(d_1 + d_2)} \sqrt{\frac{\mu}{k\phi c_i}} \quad d_1 + d_2 = 8.133 \frac{\Delta q B}{hm_{ch}} \sqrt{\frac{\mu}{k\phi c_i}} \quad \text{ft} \quad \frac{d_1}{d_1 + d_2} = \frac{1}{\pi} \text{Arc sin} \left(\frac{d_1 + d_2}{2\pi r_w} \right) e^{-S_{ch}}$$

STRAIGHT LINE METHOD FOR INTERSECTING BOUNDARIES (Wedge)

(Late Times)



Specialised Plot



Horner Plot

$$m_{\text{wedge}} = \frac{360}{\theta} m$$

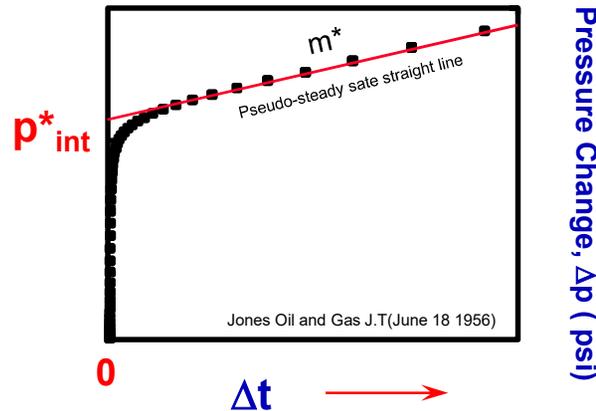
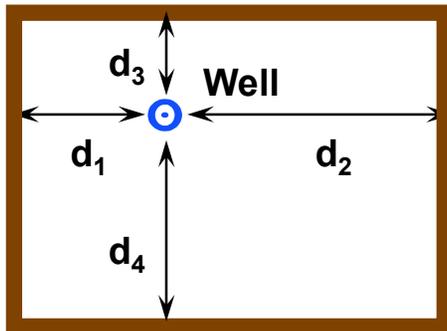
psi/(log cycle)

$$\theta^\circ = 360 \frac{m}{m_{\text{wedge}}}$$

degree

STRAIGHT LINE METHOD FOR CLOSED RESERVOIRS (Late Times)

DRAWDOWN ONLY



Specialised Plot

Horner Plot

$$\Delta p = 0.234 \frac{\Delta q B}{\phi c_i h A} \Delta t + 162.6 \frac{\Delta q B \mu}{kh} \left[\log \frac{A}{r_w^2} - \log(C_A) + 0.351 + 0.87S \right]$$

NONE

↑
Dietz shape factor

$$m^* = 0.234 \frac{\Delta q B}{\phi c_i h A} \quad \text{psi/(hr)}$$

$$\phi h A = 0.234 \frac{\Delta q B}{c_i m^*} \quad \text{cu.ft}$$

$$\Delta p_{int}^* = p_i - p_{int}^* = 162.6 \frac{\Delta q B \mu}{kh} \left[\log \frac{A}{r_w^2} - \log(C_A) + 0.351 + 0.87S \right]$$

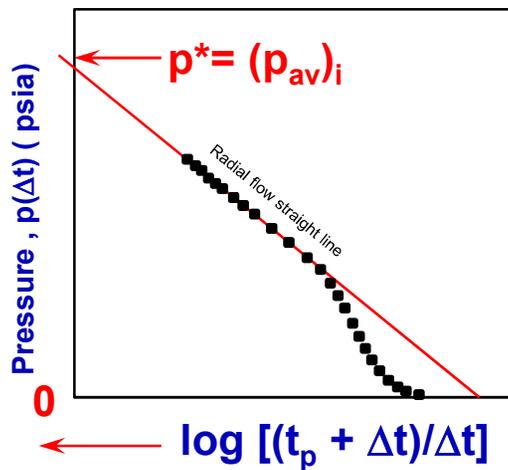
$$C_A = 5.456 \frac{m}{m^*} e^{\left(2.303 \frac{p_{1hr} - p_{int}^*}{m} \right)}$$

IMPORTANT DEFINITIONS

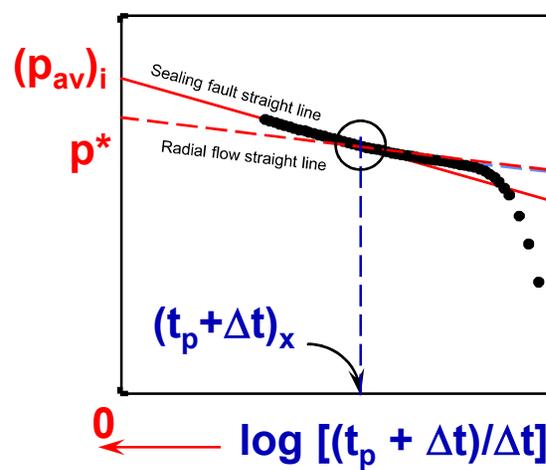
p^* IS THE INTERCEPT OF THE HORNER (SUPERPOSITION) STRAIGHT LINE

→ p^* REPRESENTS $(p_{av})_i$ IF AND ONLY IF THE RESERVOIR IS OF INFINITE EXTENT

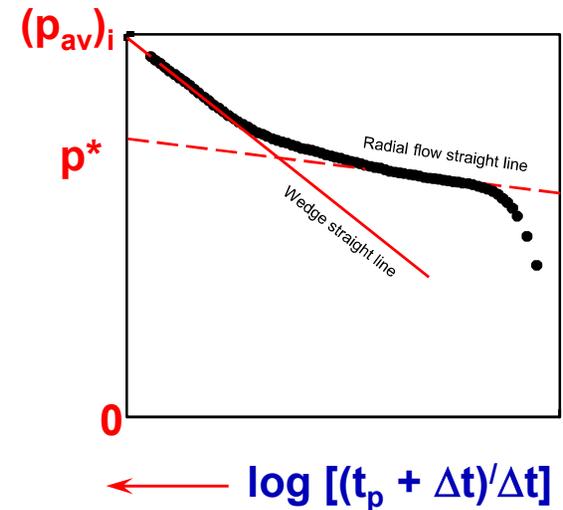
Infinite-Acting Radial Flow



Single Sealing Fault

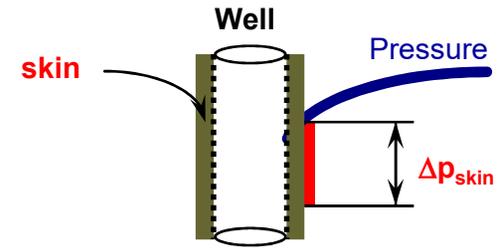


Wedge



Pressure, $p(\Delta t)$ (psia)

SKIN EFFECT (general) $p_D(t_D, S) = p_D(t_D, S = 0) + S$



(1) Skin can be calculated from specialised analyses

S is obtained from the straight line intercept

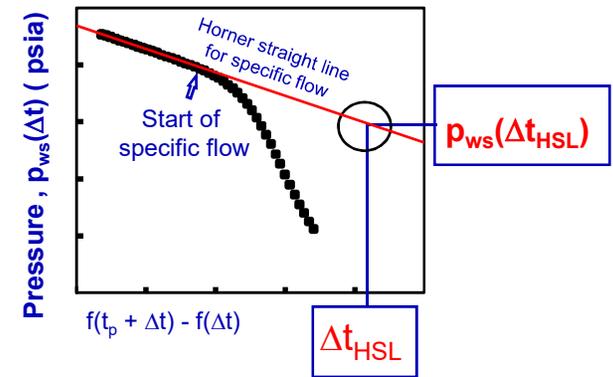
(2) Skin can be calculated from Horner analyses:

S does not appear in the Horner equation so we must make it appear:

$$PM [\bar{p}_i - p_{ws}(\Delta t)] \equiv \{p_D[\text{TM}(t_p + \Delta t)] + S\} - \{p_D[\text{TM} \Delta t] + S\}$$

$$p_{ws}(\Delta t) \equiv \bar{p}_i - \frac{1}{PM} p_D[\text{TM}(t_p + \Delta t)] + \frac{1}{PM} p_D(\text{TM} \Delta t)$$

- Select an **arbitrary** value Δt_{HSL} of the shut-in time.
- Read **on the Horner straight line** the corresponding pressure value $p_{ws}(\Delta t_{HSL})$



$$p_{ws}(\Delta t_{HSL}) \equiv \bar{p}_i - \frac{1}{PM} p_D[\text{TM}(t_p + \Delta t_{HSL})] + \frac{1}{PM} p_D(\text{TM} \Delta t_{HSL})$$

- Calculate the pressure at the end of the drawdown, $p_{wf}(t_p)$
(This is also the pressure at the time of shut-in, $p_{ws}(\Delta t = 0)$):

$$\Delta p_{wf}(t_p) = \bar{p}_i - p_{wf}(t_p) = \bar{p}_i - p_{ws}(\Delta t = 0) \equiv \frac{1}{PM} \{p_D[\text{TM}(\Delta t = 0)] + S\}$$

- Take the difference:

$$S = PM [p_{ws}(\Delta t_{HSL}) - p_{ws}(\Delta t = 0)] + p_D[\text{TM}(t_p + \Delta t_{HSL})] - p_D(\text{TM} t_p) - p_D(\text{TM} \Delta t_{HSL})$$

SKIN EFFECT FROM RADIAL FLOW

Radial flow: $p_D(t_D, S = 0) = 1.151 (\log t_D + 0.35)$ $p_D = \frac{kh}{141.2 \Delta q B \mu} \Delta p$ $t_D = \frac{0.000264 k}{\phi \mu c_t r_w^2} \Delta t$

(1) Specialised analysis: $S = 1.151 \left(\frac{\Delta p_{1 \text{ hr}}}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + 3.23 \right)$

(2) Horner analysis:

$$S = PM [p_{ws}(\Delta t_{HSL}) - p_{ws}(\Delta t = 0)] + p_D [TM(t_p + \Delta t_{HSL})] - p_D(TM t_p) - p_D(TM \Delta t_{HSL})$$

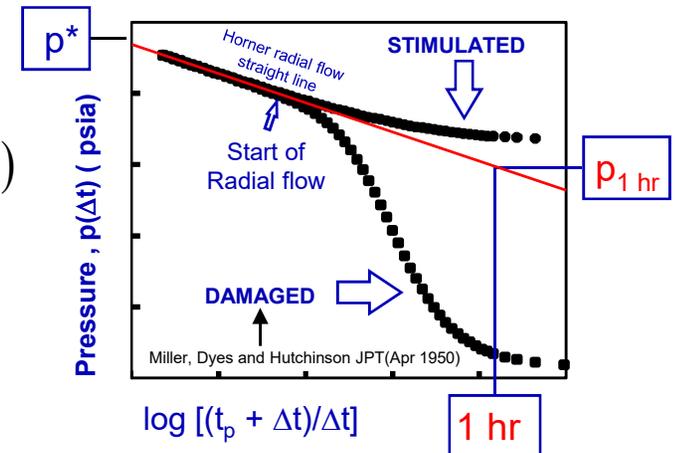
$$S = PM [p_{ws}(\Delta t_{HSL}) - p_{ws}(\Delta t = 0)] + 1.151 \log \frac{t_p + \Delta t_{HSL}}{t_p} - 1.151 \left(\log \frac{0.000264 k \Delta t_{HSL}}{\phi \mu c_t r_w^2} + 0.35 \right)$$

$$\Delta p = \frac{1.151}{PM} (\log t_D + 0.35) = m (\log t_D + 0.35) \Rightarrow PM = \frac{1.151}{m}$$

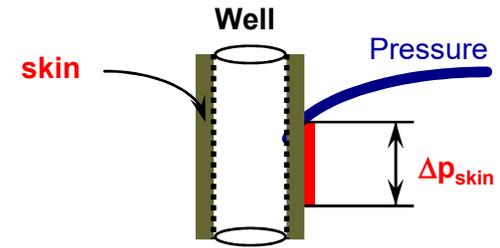
Substitute $PM = \frac{1.151}{m}$ and $\Delta t_{HSL} = 1 \text{ hour}$ so that $(\log \Delta t_{HSL} = 0)$

$$S = 1.151 \left(\frac{p_{1 \text{ hr}} - p(\Delta t = 0)}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + \log \frac{t_p + 1}{t_p} + 3.23 \right)$$

Negligible



SKIN EFFECT (general) $p_D(t_D, S) = p_D(t_D, S = 0) + S$



(1) Skin can be calculated from specialised analyses

S is obtained from the straight line intercept

(2) Skin from superposition analyses:

$$(\Delta p)_{GH} = PM[p_i - p(\Delta t)] = \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t \right) - p_D(\Delta t)_D$$

$$\text{vs. } \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} f \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t \right) - f(\Delta t)$$

$$p(\Delta t) = p_i - \frac{1}{PM} \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t \right) + \frac{1}{PM} p_D(\Delta t)_D$$

- Select an **arbitrary** value Δt_{HSL} of the shut-in time.

- Read **on the superposition straight line** the corresponding pressure value $p_{ws}(\Delta t_{HSL})$

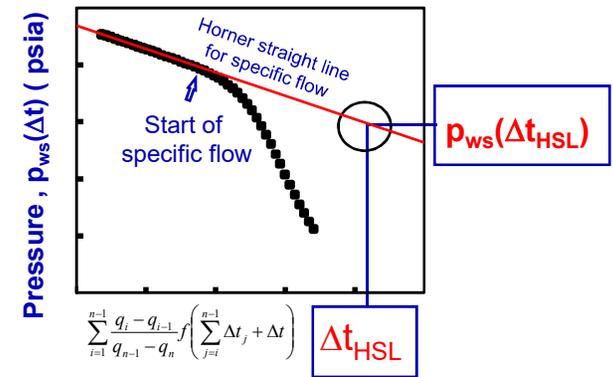
$$p(\Delta t_{HSL}) = p_i - \frac{1}{PM} \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t_{HSL} \right) \right] + \frac{1}{PM} p_D [TM(\Delta t_{HSL})]$$

- Calculate the pressure at the end of the drawdown, $p_{wf}(t_p)$
(This is also the pressure at the time of shut-in, $p_{ws}(\Delta t=0)$):

$$p(\Delta t = 0) = p_i - \frac{1}{PM} \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j \right) \right] - \frac{S}{PM}$$

- Take the difference:

$$S = PM[p(\Delta t_{HSL}) - p(\Delta t = 0)] + \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t_{HSL} \right) \right] - \sum_{i=1}^{n-1} \frac{q_i - q_{i-1}}{q_{n-1} - q_n} p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j \right) \right] - p_D [TM(\Delta t_{HSL})]$$



SKIN EFFECT FROM RADIAL FLOW

Radial flow: $p_D(t_D, S = 0) = 1.151 (\log t_D + 0.35)$ $p_D = \frac{kh}{141.2 \Delta q B \mu} \Delta p$ $t_D = \frac{0.000264 k}{\phi \mu c_t r_w^2} \Delta t$

(1) Specialised analysis:

$$S = 1.151 \left(\frac{\Delta p_{1 \text{ hr}}}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + 3.23 \right)$$

(2) Superposition analysis:

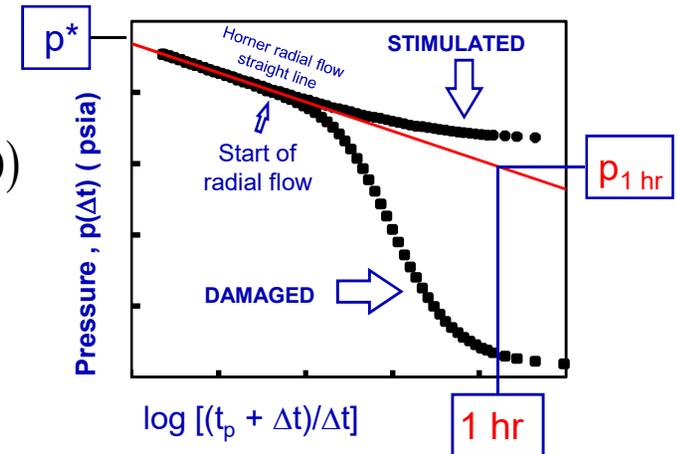
$$S = PM [p(\Delta t_{HSL}) - p(\Delta t = 0)] + \frac{1}{q_{n-1} - q_n} \left\{ \sum_{i=1}^{n-1} (q_i - q_{i-1}) p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t_{HSL} \right) \right] - \sum_{i=1}^{n-1} (q_i - q_{i-1}) p_D \left[TM \left(\sum_{j=i}^{n-1} \Delta t_j \right) \right] \right\} - p_D [TM(\Delta t_{HSL})]$$

$$S = 1.151 \left[\frac{p(\Delta t_{HSL}) - p(\Delta t = 0)}{m'(q_{n-1} - q_n)} + \sum_{i=1}^{n-1} \frac{(q_i - q_{i-1})}{q_{n-1} - q_n} \log \frac{\left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t_{HSL} \right)}{\sum_{j=i}^{n-1} \Delta t_j} - \log \frac{0.000264 k \Delta t_{HSL}}{\phi \mu c_t r_w^2} - 0.35 \right]$$

Substitute $PM = \frac{1.151}{m'(q_{n-1} - q_n)}$ and $\Delta t_{HSL} = 1 \text{ hour}$ so that $(\log \Delta t_{HSL} = 0)$

$$S = 1.151 \left[\frac{p(1 \text{ hr}) - p(\Delta t = 0)}{m'(q_{n-1} - q_n)} - \log \frac{k}{\phi \mu c_t r_w^2} + \sum_{i=1}^{n-1} \frac{(q_i - q_{i-1})}{q_{n-1} - q_n} \log \frac{\left(\sum_{j=i}^{n-1} \Delta t_j + 1 \right)}{\sum_{j=i}^{n-1} \Delta t_j} + 3.23 \right]$$

Negligible



TOTAL SKIN EFFECT

$$S_{Total} = 1.151 \left(\frac{p_{1hr} - p_{ws}(\Delta t = 0)}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + \log \frac{t_p + 1}{t_p} + 3.23 \right)$$

S_{Total} includes :

$$S_{Mechanical} \text{ (damaged, stimulated)} + S_{Fluid} \text{ (Gas, multiphase, fluid bank)} + S_{Completion} \text{ (fractured well, limited entry, inclined well)} + S_{Geology} \text{ (anisotropy, natural fractures)}$$

$$S_{Mechanical} = -4 \text{ (acidised)} \rightarrow +20 \text{ (damaged)}$$

$$S_{Gas} = +5 \rightarrow +20 \text{ (= } Dq \text{)}$$

$$S_{Multiphase} = +5 \rightarrow +20$$

$$S_{Anisotropy} = -2 \rightarrow 0$$

$$S_{Completion} = -5.5 \text{ (Fractured or horizontal well)} \rightarrow$$

$$S_{Fluid Bank} = +2 \text{ (Gas to oil/water)} \rightarrow >20 \text{ (Condensate to gas)}$$

$$S_{Geology} = -3 \text{ (geoskin in fissured reservoirs)} \rightarrow 0$$

$$\left. \begin{array}{l} S_{Mechanical} \\ S_{Gas} \\ S_{Multiphase} \\ S_{Anisotropy} \end{array} \right\} S_w = -4 \rightarrow 60$$

$$\left. \begin{array}{l} S_{Completion} \\ S_{Fluid Bank} \\ S_{Geology} \end{array} \right\} \text{Included in the interpretation model}$$

$$\text{Well with limited entry: } S_{pp} = \frac{h}{h_w} S_w + S_{Completion}$$

$$\text{if } \frac{h}{h_w} = 20 \text{ and } \frac{k_z}{k_r} = 0.1, S_{Completion} \approx 100$$

$$\text{if } S_w = 10, S_{pp} = 300$$

CONCEPT OF EFFECTIVE WELLBORE RADIUS

$$r_{we} = r_w e^{-S}$$

Damaged well: $r_{we} < r_w$

Stimulated well: $r_{we} > r_w$

A dimensionless time based on the effective wellbore radius is used to represent:

- the semi-log radial flow regime
- by the same equation
- for all models,

independently of the near-wellbore conditions:

$$p_D = 1.151(\log t_{De} + 0.35) \quad p_D = \frac{1}{2}(\ln t_{De} + 0.80907) \quad t_{De} = \frac{0.000264 k}{\phi \mu c_t r_{we}^2} \Delta t$$

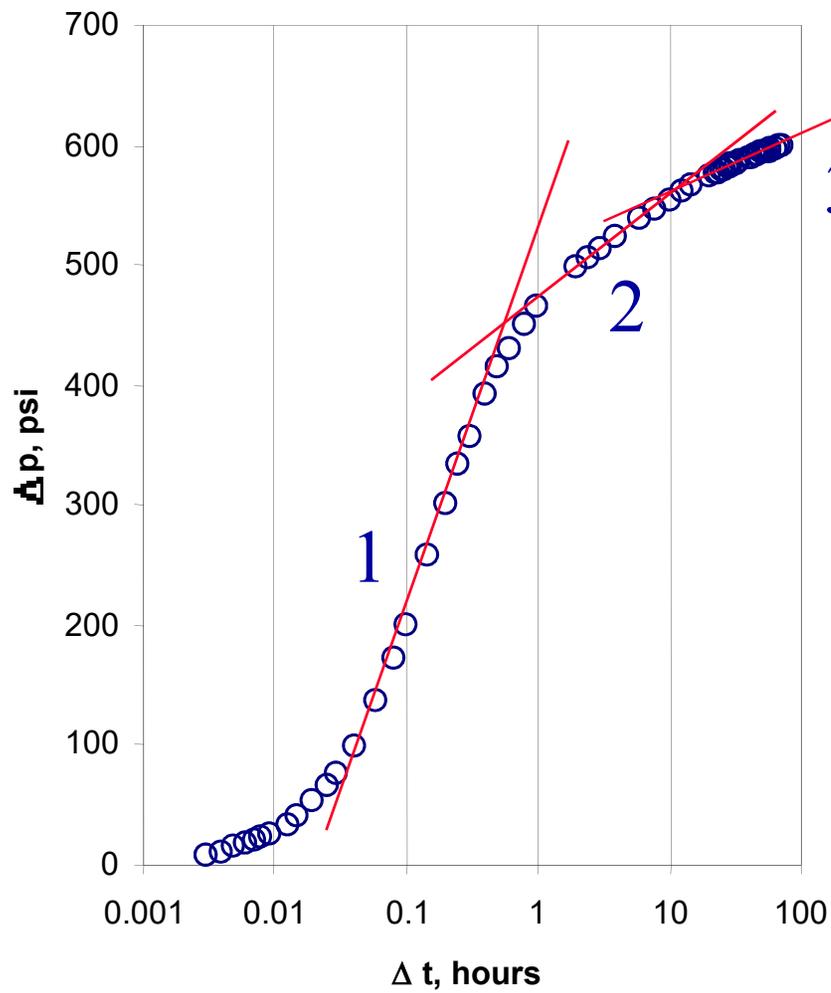
High conductivity fracture:

$$r_{we} = \frac{x_f}{2}$$

Low conductivity fracture:

$$r_{we} = f(x_f, k_f w_f)$$

Example 1: MDH plot



Choice	1	2	3
Class			
m			

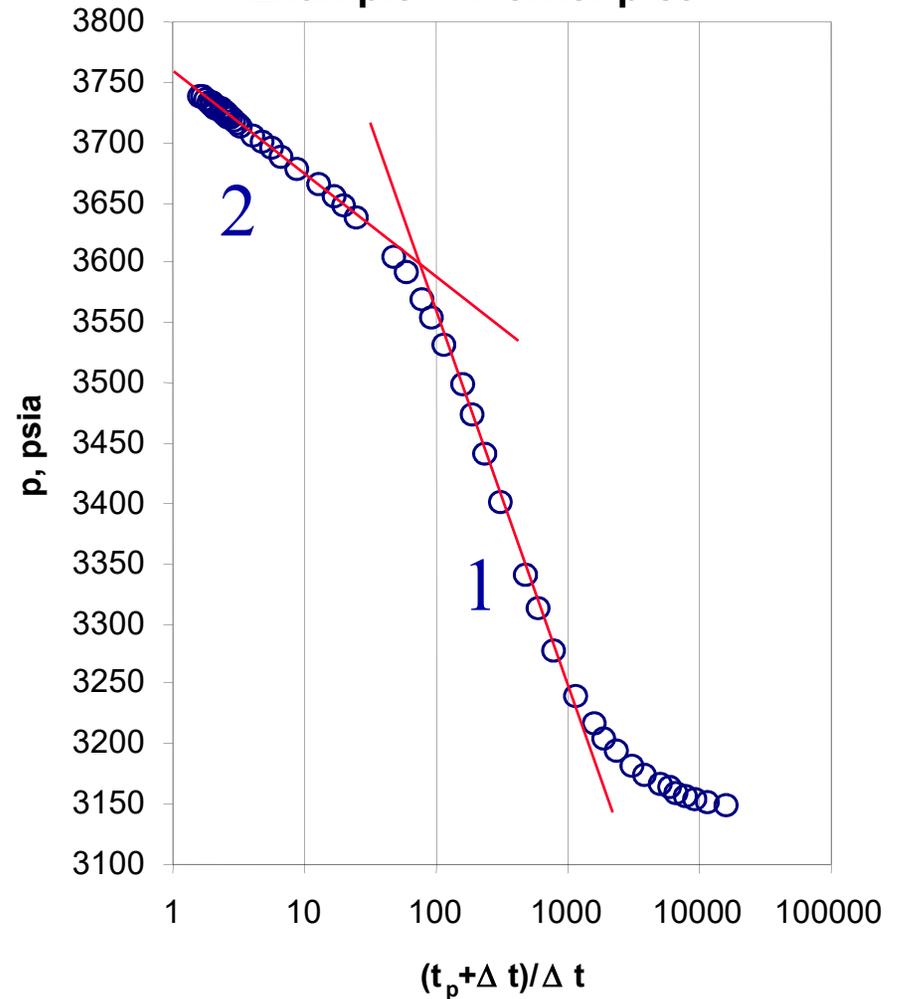
Horner 3rd World Pet. Congress (1951)

In the application of these methods there are certain definite dangers. Firstly it is not always clear which part of the build-up curve is to be used to determine k . It is not uncommon for many of the early pressure readings to fall on a straight line, when plotted against $\log_{10} \frac{t_0 + \theta}{\theta}$ although they have been taken during the period of the after-production.

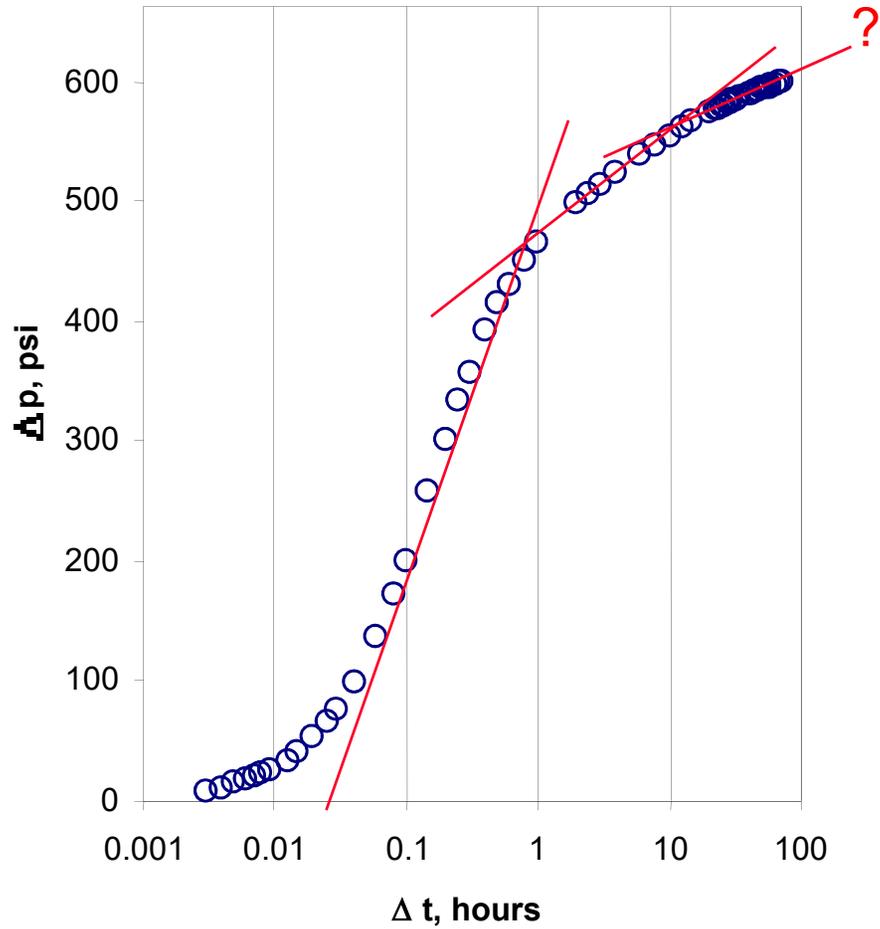
After analysing a large number of wells by this method we have obtained the impression that acceptable values for the permeability are more frequently obtained than for the extrapolated pressure. This is probably due to the fact that whereas one is content with only an approximate value of k the limits of error within which the reservoir pressure is required are very much smaller.

Choice	1	2	
Class			
m			

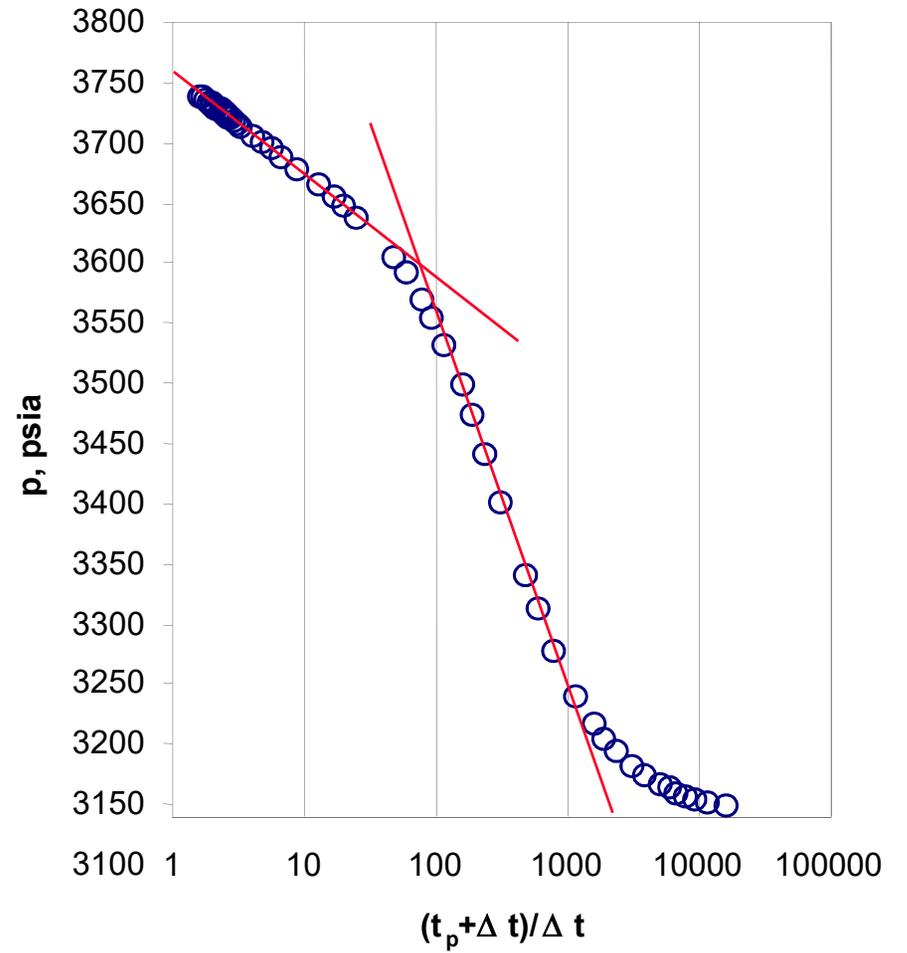
Example 1: Horner plot



Example 1: MDH plot



Example 1: Horner plot



DIFFERENCE BETWEEN MDH AND HORNER PLOTS

Perrine - Analysis of pressure build up curves API-1956-482

Horner analysis is applied to new wells where only a small fraction of the oil in place has been produced.

MDH analysis is applicable to wells where the effect of a drainage boundary has been felt at the well (pseudo-steady state reached during the drawdown).

LIMITATIONS OF SPECIALISED PLOTS

INFLUENCE OF PRODUCTION TIME

Ramey and Cobb SPE 3012 45th ATCE(Oct 1970)

A careful study of Horner graph for a wide range of producing periods (from very short to well into pseudosteady state) reveals that it *does* always present a straight line of the proper slope. But more important, the Homer method straightens buildup data to much longer shut-in times than does a Miller-Dyes-Hutchinson graph.

An attempt to place a straight line through buildup data for a similar time period on a Miller-Dyes-Hutchinson graph would lead to a lower slope and an erroneously high estimate of permeability by the Miller-Dyes-Hutchinson graph.

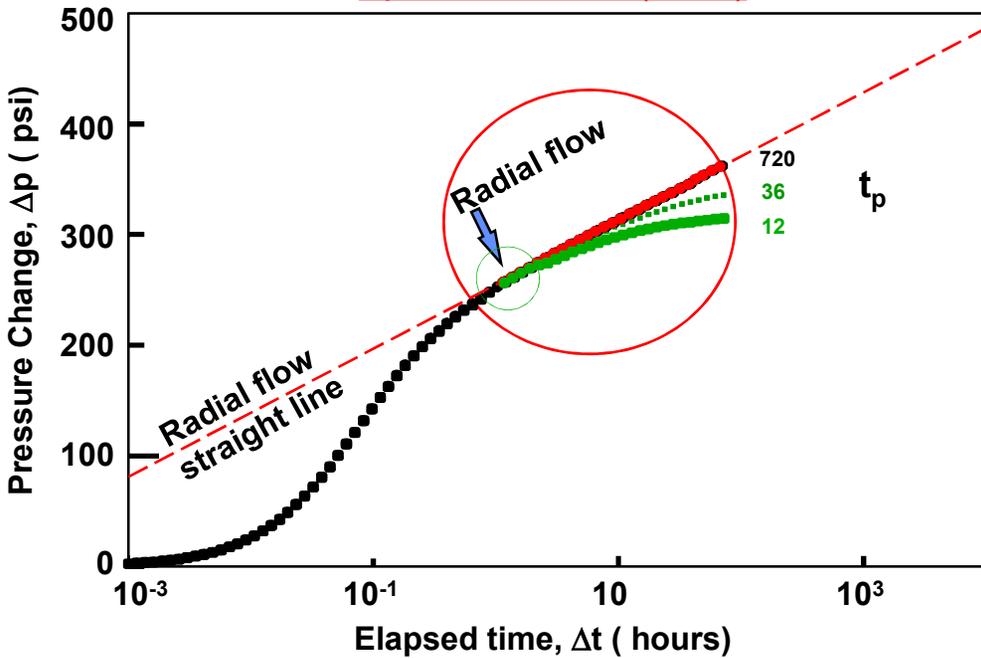
It is apparent that identification of the proper straight line could be difficult for a Miller-Dyes-Hutchinson graph for short producing times.

This explains a puzzling difference in permeability often noticed in field operations when both MillerDyes-Hutchinson and Horner graphs are constructed for a single set of data. The Horner method appears far more reliable than the Miller-Dyes-Hutchinson method. This empirical observation is perhaps one of the most important results of our study.

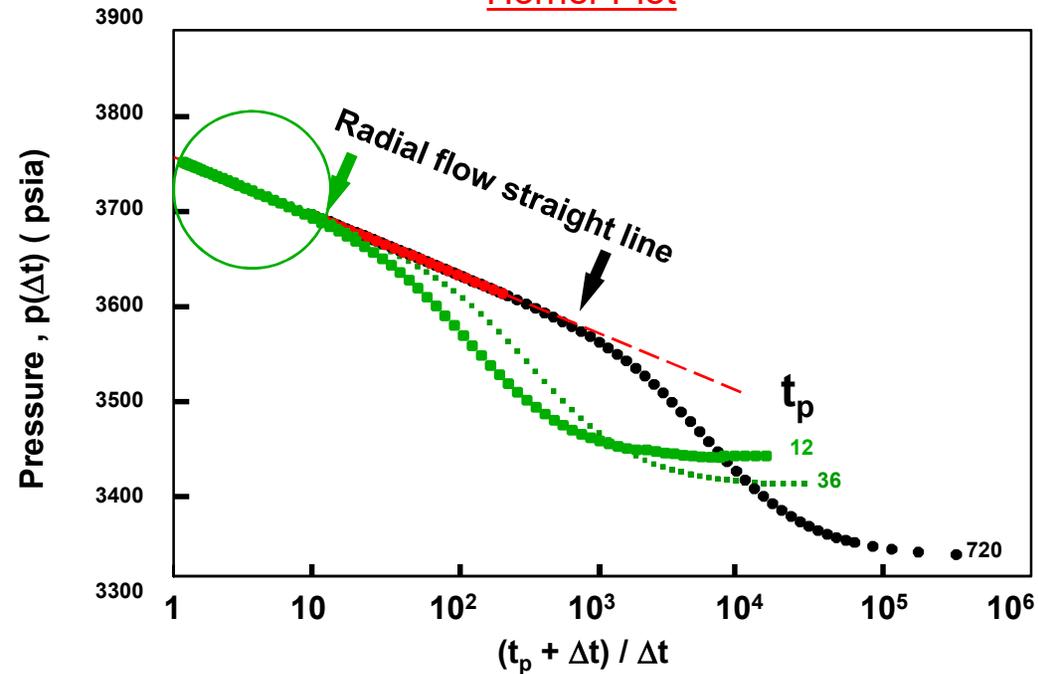
LIMITATIONS OF SPECIALISED PLOTS

INFLUENCE OF PRODUCTION TIME

Specialised Plot (MDH)



Horner Plot



$$p_{\text{BU}D}(\Delta t)_D = p_D(\Delta t)_D + p_D(t_p)_D - p_D(t_p + \Delta t)_D$$

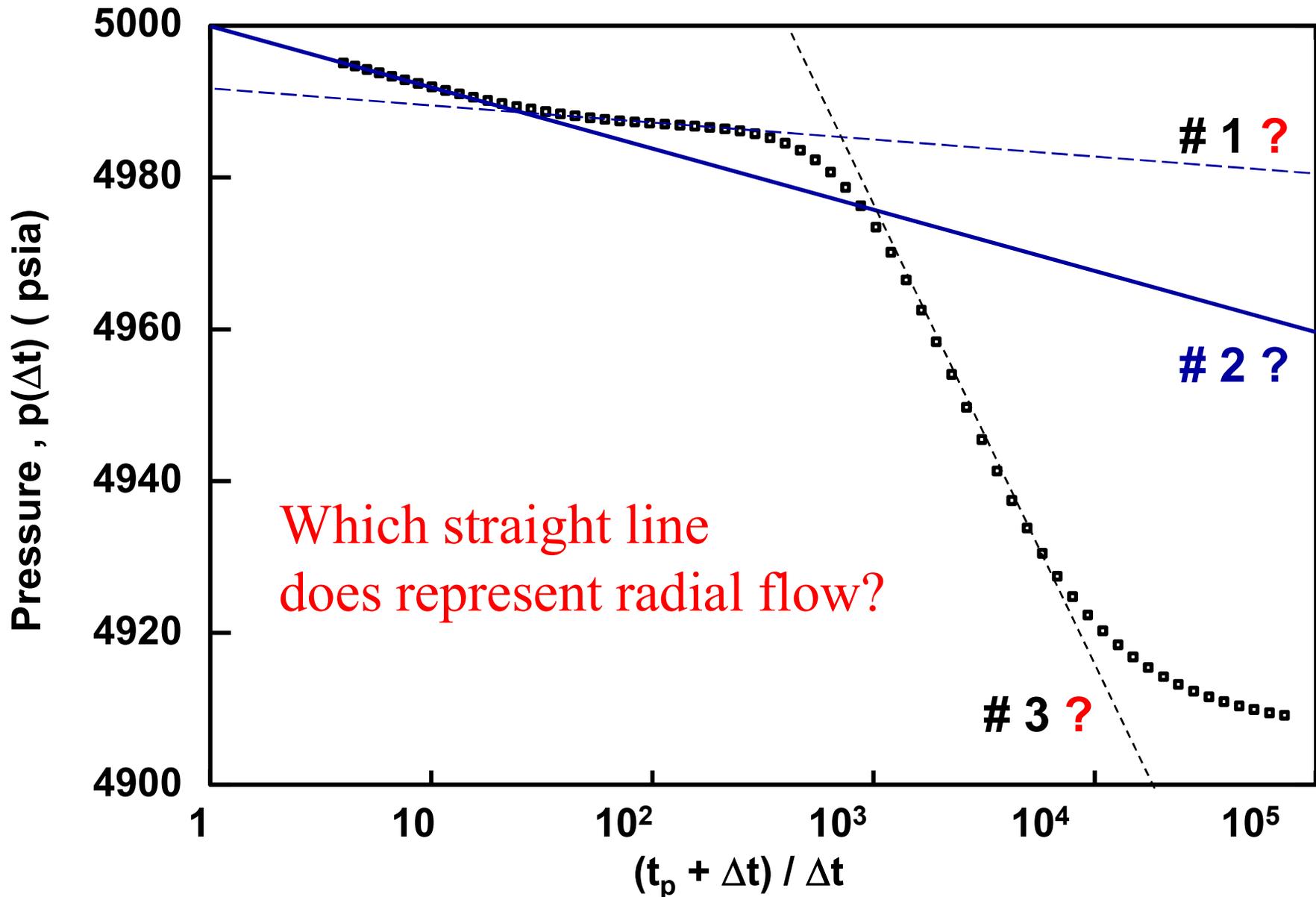
$$p_{\text{BU}D}(\Delta t)_D = p_D(\Delta t)_D \propto \log(\Delta t) \text{ if and only if}$$

$$p_D(t_p)_D - p_D(t_p + \Delta t)_D \text{ can be neglected}$$

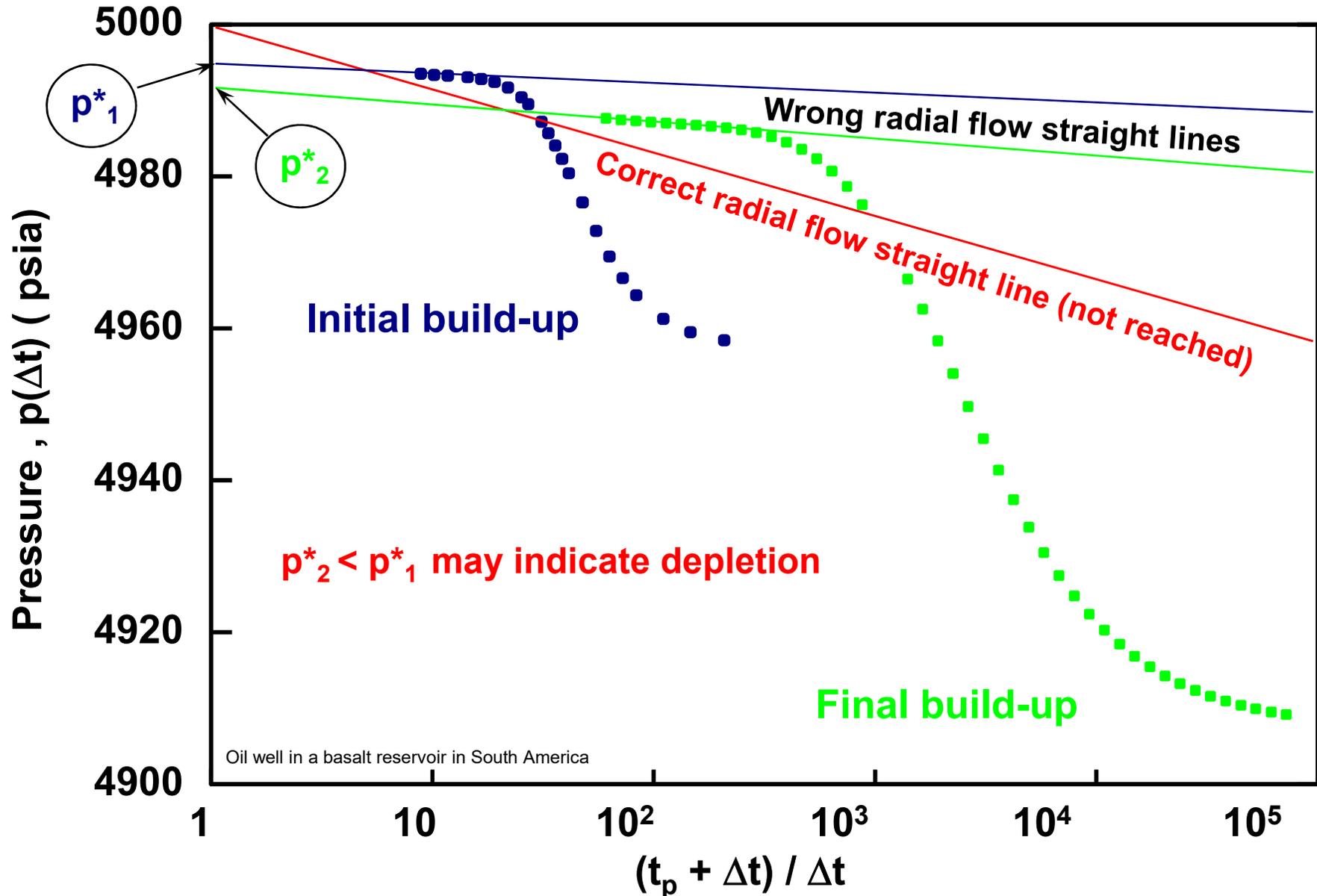
$$p_{\text{H}D}(\Delta t)_D = p_D(t_p + \Delta t)_D - p_D(\Delta t)_D$$

$$\propto \log(t_p + \Delta t)_D - \log(\Delta t)_D = \log \frac{t_p + \Delta t}{\Delta t}$$

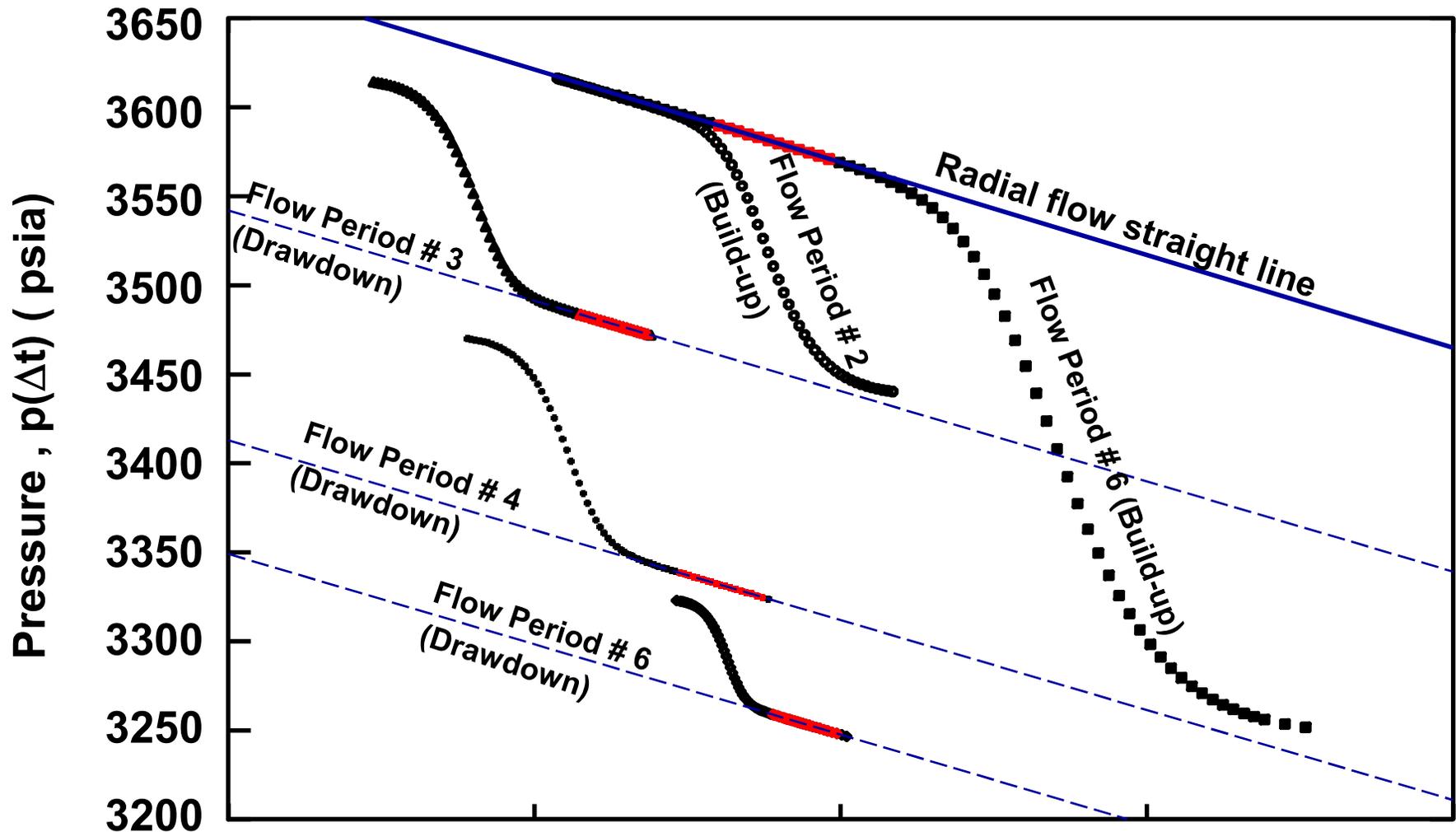
LIMITATIONS OF STRAIGHT LINE METHODS IDENTIFICATION



LIMITATIONS OF STRAIGHT LINE METHODS IDENTIFICATION

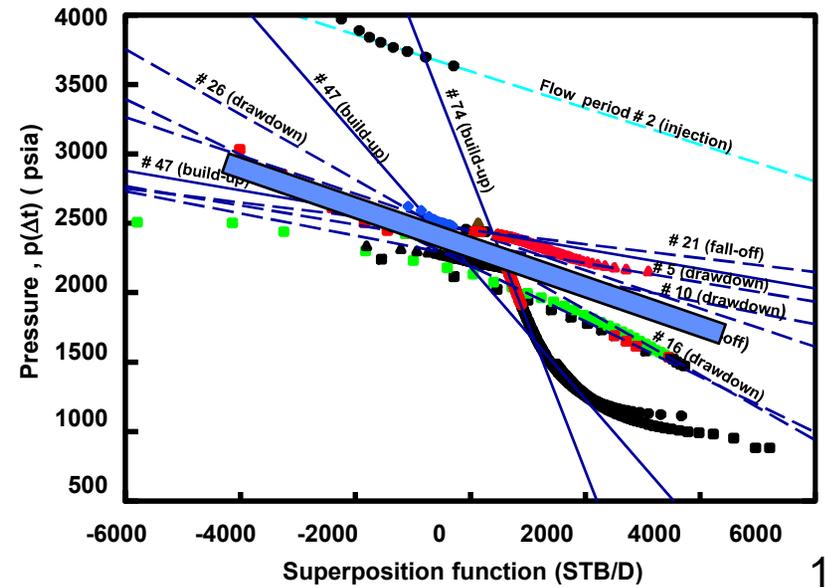
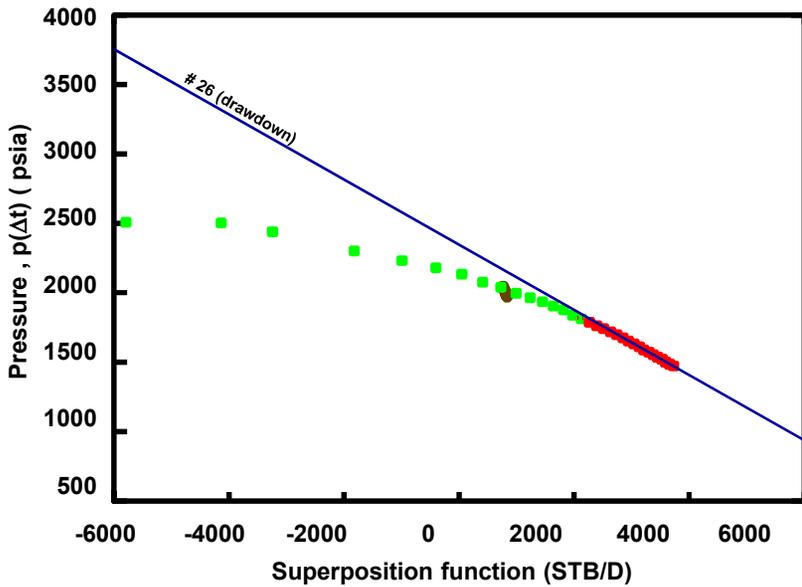
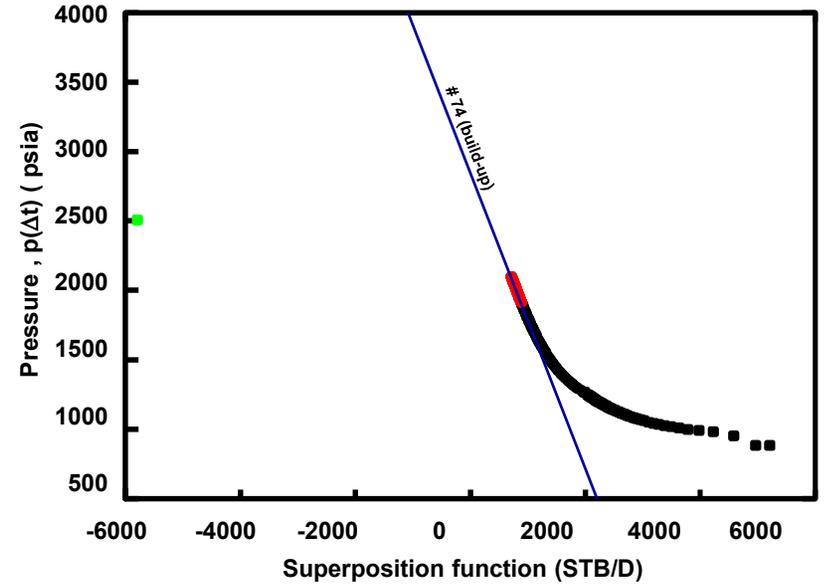
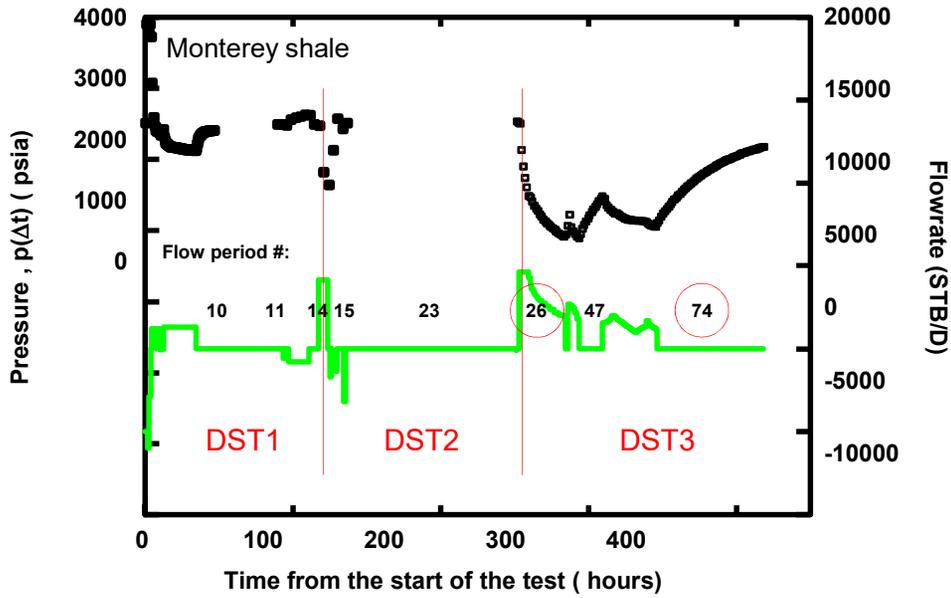


VERIFICATION



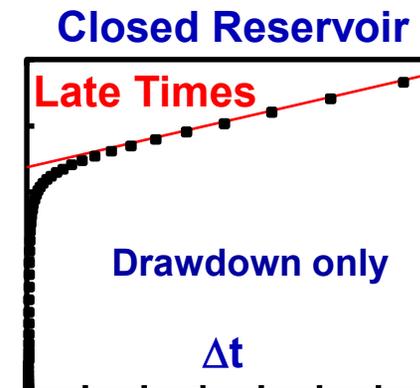
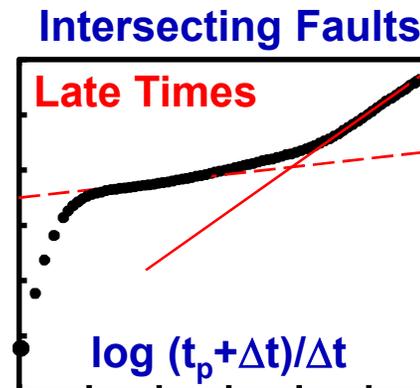
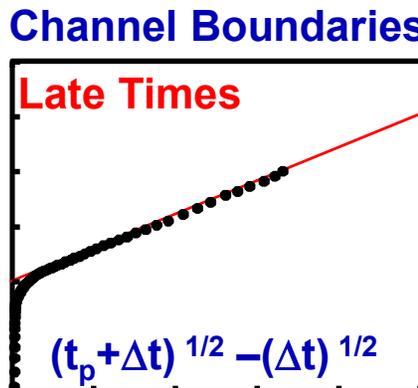
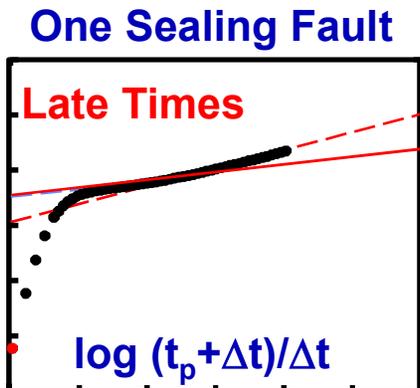
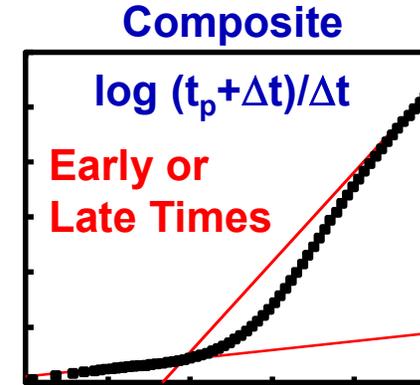
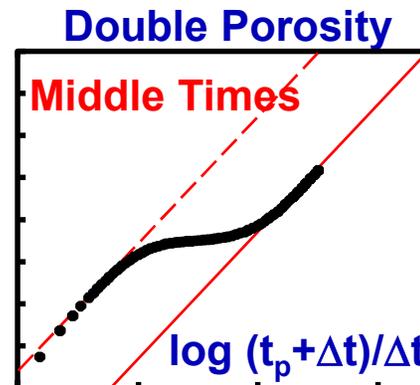
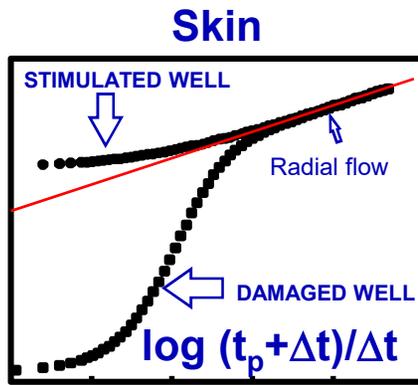
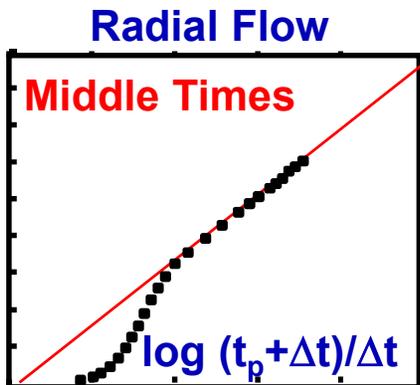
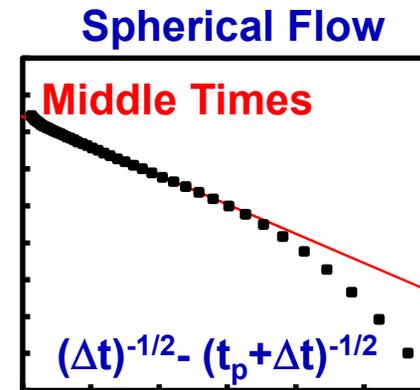
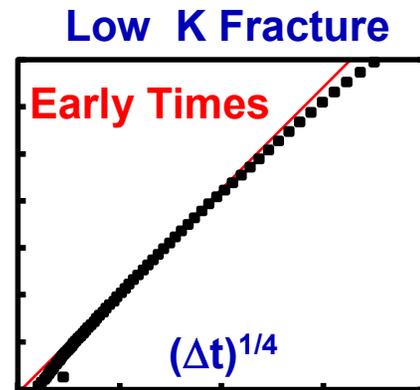
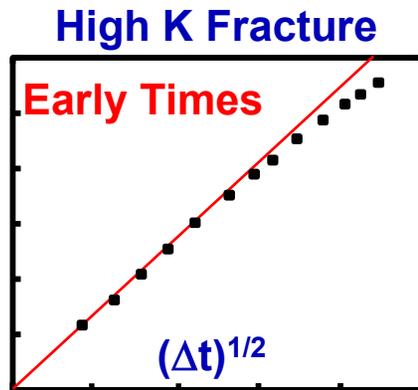
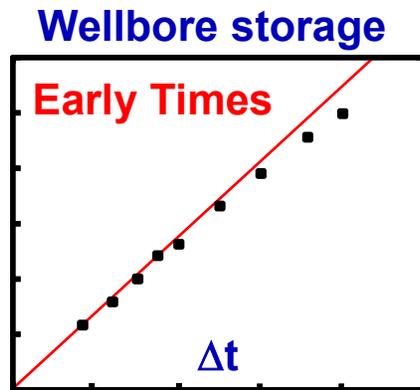
$$\sum_{i=1}^{n-1} (q_i - q_{i-1}) \log \left(\sum_{j=i}^{n-1} \Delta t_j + \Delta t \right) - (q_{n-1} - q_n) \log(\Delta t) \quad (\text{STB/D})$$

EXAMPLE



Summary: STRAIGHT LINE METHODS

Pressure Change, p or Δp (psi)



Flow Regime Specific Function of Elapsed Time

Summary: STRAIGHT LINE METHODS

ADVANTAGES:

- Simple, Easy to Implement

LIMITATIONS:

- Difficult to select the proper straight line
- Length of straight line function of production time
- Flow regime may exist even if straight line does not (2-porosity)
- no straight line, no analysis
- no validation (except with multi flow periods)